Problem 1

1. Let $v(t) = \sqrt{2} V \cos(\omega t + \theta_v)$, $i(t) = \sqrt{2} I \cos(\omega t + \theta_i)$

\[ p(t) = \frac{V I}{\text{dc}} \cos(\theta_v - \theta_i) + \frac{V I}{\text{dc}} \cos(2\omega t + \theta_v + \theta_i) \]

The DC component is due to $R$. \[ \langle p(t) \rangle = \frac{1707 - 293}{2} = 707 \text{ (average power)} \]

We can also let $\theta_v = 0$ as reference.

\[ \therefore 1000 \cos(-\theta_i) = 707 \text{ (From DC component)} \]

The max power is $1707 \text{ W} \Rightarrow V I \cos(2\omega t + \theta_v + \theta_i) = 1000 \text{ W}$ for some $t$ such that power is max. This is when $\cos(2\omega t + \theta_v + \theta_i) = 1$

\[ \therefore V I = 1000 \text{ W} \Rightarrow I = 10 \text{ A} \]

\[ \theta_i = -0.785 \text{ rad} = -45^\circ \]

\[ Z = \frac{V}{I} = \frac{100}{10 \angle -45^\circ} = 7.07 + j7.07 \Omega \]

\[ \therefore R = 7.07 \Omega \quad L = \frac{7.07}{\omega} \text{ H} \]

2. $S = V I^* = 100 \angle 0^\circ \cdot 10^\medcirc \angle 45^\medcirc = 1000 \angle 45^\medcirc = 707 + j707 \text{ VA}$

3. Max power into $L$ is equal to $Q$:

\[ 707 \text{ VAR} \]

see "Physical Significance of Real and Reactive Power" in text book. Pg. 52 in 5th edition
Problem 2

Power triangle: \[ S \]

\[ \cos(\Theta) = 0.7 \Rightarrow \Theta = \cos^{-1}(0.7) = 0.795 \]

Also, we know \( \Theta = \Theta_{V} - \Theta_{I} > 0 \) since it's "lagging"

\[ Q = S \cdot \tan(\Theta) = 5.10 \text{ MVAR} \]

Need \( pf = 0.9 \Rightarrow \Theta' = \cos^{-1}(0.9) \)

\[ Q' = S \cdot \tan(\Theta') = 2.42 \text{ MVAR} \]

Reactive Power Required is \( Q' - Q = -2.68 \text{ MVAR} \)

Note: We know \( P \) (real power) doesn't change because ideal inductors and capacitors do not absorb real power. \( Q_{\text{cap}} \) is negative since capacitors absorb negative reactive power (delivers positive reactive power)

See Example 2.10 of ECE 330 book for similar problem.
Problem 3

Per phase: \( \frac{200}{3} = 66.67 \text{ KW at } pf = 0.707 \text{ lag} \)
\[ \frac{50}{3} = 16.67 \text{ KVAR} \]

Load:

\[ Q = \tan(\theta) \cdot 66.67 = \tan (\cos^{-1}(0.707)) \cdot 66.67 = 66.69 \text{ KVAR} \]

Cap. Bank: \( Q_c = 16.67 \text{ KVAR} \)

\[ Q' = Q + Q_c = 66.69 - 16.67 = 50.02 \text{ KVAR} \]
\[ P' = P = 66.67 \text{ KW} \]
\[ \tan (\theta') = \frac{Q'}{P'} = 0.644 \]
\[ \cos (\theta') = 0.799 \text{ lag} \]

\[ S = VI^* \Rightarrow |I| = \frac{|I|}{|V|} = \frac{\sqrt{66.67^2 + 50.02^2}}{440} = 1.89 \text{ A per phase} \]

Note: Credit was also given for \( V = \frac{440}{\sqrt{3}} \) since it wasn't clear if 440 was \( V \), \( I \), or \( I_l \).

In this case, \( |I| = \frac{151}{440} = 0.328 \text{ A per phase} \)
Problem 4

Power triangle:

\[ \cos(\theta) = 0.9 \quad \theta > 0 \]

\[ \tan(\theta) \cdot 10 = 4.84 \text{ KVAR} \]

1. \[ S = 10 \text{ kW} + j 4.84 \text{ KVAR} \]

2. \[ |I| = \frac{|S|}{\text{IV}} = 26.7 \text{ A} \]

3. \[ p(t) = P(1 + \cos(2\omega t + \phi_1)) - Q \sin(2\omega t + \phi_2) \quad \text{from pg. 15 ECE330 Book} \]

\[ = 10000(1 + \cos(2\omega t)) - 4840 \sin(2\omega t) \]
Problem 5

\[ I = j(0.5 \times 2) = j \]

\[ V_s \]

\[ 480 \angle 0^\circ \]

\[ 120 \text{ kW}, 0.85 \text{ lagging} \]

1. \[ S_s = S_L + S_{\text{plant}} \]

\[ Q = 120 \cdot \tan(\cos^{-1}(0.85)) = 74.37 \text{ KVAR} \]

\[ S_{\text{plant}} = V_{\text{plant}} I^* = 141.18 \angle 0.555 \text{ KVA} \]

\[ \angle \theta^* = -\cos(0.85) = -0.555 \]

\[ |I| = \frac{|S_{\text{plant}}|}{|V_{\text{plant}}|} = 294 \text{ A} \Rightarrow I = 294 \angle -0.555 \]

\[ S_L = |\vec{I}|^2 = j86.43 \text{ KVA} \]

\[ S_s = 120 + j74.37 + j86.43 = 120 + j160.8 \text{ KVA} = 200.6 \angle 0.929 \text{ KVA} \]

\[ V_s = \frac{S_s}{I^*} = \frac{200.6 \angle 0.929 \text{ KVA}}{294 \angle 0.555 \text{ A}} = 682.3 \angle 0.374 \text{ V} \]

\[ 0.374 \text{ rad} \]

\[ 21.4^\circ \]

\[ \text{pf at source: } \cos(0.374 + 0.555) = 0.598 \text{ lagging} \]
Problem 5

2. Using KVL:

\[ V_s = V_L + 480 \angle 0 \]

\[ = I_L Z_L + 480 \angle 0 \]

\[ = 2.94 \angle -0.555 \cdot j + 480 \angle 0 \]

\[ = 2.94 [ \cos(1.01) + j \sin(1.01) ] + 480 \]

\[ = 154.9 + j 249.87 + 480 \]

\[ = 634.9 + j 249.87 \]

\[ = 682.3 \angle 0.375 \text{ V} \]

pf at source: \[ \cos(0.375 + 0.555) = 0.597 \text{ lagging} \]