Problem 1

5 buses ⇒ \( Y_{bs} \) is 5×5 matrix

row 3 ⇒ Need admittances connected to bus 3

\[ Y(3,3) = \text{Sum of admittances connected to bus 3} \]

\[ = \frac{1}{R_{39} + jX_{39}} \]

\[ = \frac{1}{0.00075 + j0.01} \]

\[ = 7.46 - j99.4 \text{ p.u.} \]

\[ Y(3,1) = Y(3,2) = Y(3,5) = 0 \quad (\text{No connection between these buses}) \]

\[ Y(3,4) = -Y(3,3) = -7.46 + j99.4 \text{ p.u.} \]

\[ \therefore \ Y(3,*) = \begin{bmatrix} 0 & 0 & 7.46 - j99.4 & -7.46 + j99.4 & 0 \end{bmatrix} \]

1 point per answer

Problem 2

5 points

\[ Y = \frac{1}{2} \]

\[ Y_{bus} = \begin{bmatrix}
-j8.5 & j2.5 & j5 & 0 \\
j2.5 & -j8.75 & j5 & 0 \\
j5 & j5 & -j22.5 & j12.5 \\
0 & 0 & j12.5 & -j12.5
\end{bmatrix} \]

-1 for each wrong answer in matrix
Problem 3

1 point a) See Figure above

b) Two buses ⇒ $Y_{bus}$ is $2 \times 2$, $Y' = \frac{1}{2} = 5 - j15$

2 points $Y_{bus} = \begin{bmatrix} 5-j15 & -5+j15 \\ -5+j15 & 5-j19.75 \end{bmatrix}$

c) Bus 1:

\[ P_1 = \left| \vec{V}_{01} \right| \left[ 5 \cos(\theta_1-\theta_2) - 15 \sin(\theta_1-\theta_2) \right] + \left| \vec{V}_{01} \right| \left| \vec{V}_{21} \right| \left[ -5 \cos(\theta_1-\theta_2) + 15 \sin(\theta_1-\theta_2) \right] \]

\[ Q_1 = \left| \vec{V}_{01} \right| \left[ 5 \sin(\theta_1-\theta_2) + 15 \cos(\theta_1-\theta_2) \right] + \left| \vec{V}_{01} \right| \left| \vec{V}_{21} \right| \left[ -5 \sin(\theta_1-\theta_2) - 15 \cos(\theta_1-\theta_2) \right] \]

4 points

Bus 2:

\[ P_2 = \left| \vec{V}_{2} \right|^2 \left[ 5 \cos(\theta_2-\theta_3) - 14.75 \sin(\theta_2-\theta_3) \right] + \left| \vec{V}_{2} \right| \left| \vec{V}_{01} \right| \left[ -5 \cos(\theta_2-\theta_3) + 15 \sin(\theta_2-\theta_3) \right] = 1 \]  \text{given load real power}

\[ Q_2 = \left| \vec{V}_{2} \right|^2 \left[ 5 \sin(\theta_2-\theta_3) + 14.75 \cos(\theta_2-\theta_3) \right] + \left| \vec{V}_{01} \right| \left| \vec{V}_{21} \right| \left[ -5 \sin(\theta_2-\theta_3) - 15 \cos(\theta_2-\theta_3) \right] \]

\[ Q_2 = 14.75 \left| \vec{V}_{2} \right|^2 + \left| \vec{V}_{01} \right| \left| \vec{V}_{21} \right| \left[ -5 \sin(\theta_2-\theta_3) - 15 \cos(\theta_2-\theta_3) \right] = -0.5 \]

Note: $P_2$ is '-1' because the load draws 1 p.u. of real power ⇒ injects -1 p.u. into bus 2.
Problem 4

Effects on $V_2$: With the new line, $|V_2|$ increase

Line Loadings: All other lines have % loading that decreased except for line 4-5

Real Power Losses: There's less real power losses (decreased)

For details, see table below.

<table>
<thead>
<tr>
<th></th>
<th>Before Addition of Line</th>
<th>After Addition of Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus Voltage $V_2$ (p.u.)</td>
<td>0.834</td>
<td>0.953</td>
</tr>
<tr>
<td>Total real power losses (MW)</td>
<td>34.8</td>
<td>18.3</td>
</tr>
<tr>
<td>Branch b/w bus 1-5 (% loading)</td>
<td>68.5</td>
<td>63.1</td>
</tr>
<tr>
<td>Branch b/w bus 2-4 (% loading)</td>
<td>27.3</td>
<td>17.5</td>
</tr>
<tr>
<td>Branch b/w bus 2-5 (% loading)</td>
<td>49.0</td>
<td>25.4 (both lines)</td>
</tr>
<tr>
<td>Branch b/w bus 3-4 (% loading)</td>
<td>53.1</td>
<td>45.7</td>
</tr>
<tr>
<td>Branch b/w bus 4-5 (% loading)</td>
<td>18.8</td>
<td>22.1</td>
</tr>
</tbody>
</table>