WIND FARM MODEL FOR POWER SYSTEM STABILITY ANALYSIS

BY

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DISSERTATION

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In this thesis, the modeling of wind farms based on Type-C Wind Turbine Generators (WTGs) is studied. Based on time scale decomposition, two detailed dynamic models are presented. In both models, a rotor speed controller, a reactive power controller and a pitch angle controller are considered. The turbine’s aerodynamic is represented by a static model and a single-mass model is assumed. With respect to the controllers, the speed controller is designed to extract maximum power from the wind for a given wind speed. The reactive power controller is designed to follow a reference. The pitch angle controller is designed to limit the maximum active power output. All controllers use proportional and integral control. Modal and bifurcation analysis is performed revealing that the WTG’s variables do not exhibit major oscillatory behavior when the system is perturbed. Moreover, the WTG’s variables do not participate in unstable modes and they do not change the system stability structure. In general, the most important interaction between WTGs and the system is the interchange of power. An aggregated model is proposed for wind farms. This model is characterized by a single equivalent WTG and an equivalent wind speed. Moreover, the order of the aggregated model is reduced by using selective modal analysis. This technique focuses on the most relevant modes and variables. Irrelevant variables are expressed in terms of the relevant ones, which allows reducing the model order. Replacing either a two-axis or zero-axis model of a WTG for the reduced-order model neither considerably alters the original system dynamics nor modifies the system variables. An important reduction of simulation time and model complexity is obtained. In the largest case, it is shown that two wind farms that in total are represented by 500 differential equations and 800 algebraic equations can be represented by just 4 differential equations.
To Rebeca and Mario, my beloved parents
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LIST OF ABBREVIATIONS

ACRONYMS

DAEs Differential algebraic equations
DFIG Doubly-fed induction generator
HB Hopf bifurcation
HSG Hypothetical synchronous generator
IEEE Institute of Electrical and Electronics Engineers
IM Induction motor
LP Limit point
NL Negative load
PCC Point of common coupling between a wind farm and the power system
PV Bus with controlled active power and controlled voltage in power flow studies
PQ Bus with controlled active and reactive power in power flow studies
RAC Room air-conditioner
SG Synchronous generator
SMA Selective modal analysis
SNB Saddle-node bifurcation
WTG Wind turbine generator
GREEK LETTERS

$\beta_1, \ldots, \beta_5$ Parameters of the reduced-order model of a WTG

$\delta$ Loadability angle of a SG, [rad]

$\theta$ Pitch angle of a WPG, [degree]

$\theta_D$ Terminal voltage angle of a DFIG, [rad]

$\theta_L$ Terminal voltage angle of a load, [rad]

$\theta_{\text{max}}$ Maximum pitch angle, [degree]

$\theta_S$ Terminal voltage angle of a SG, [rad]

$\lambda$ Tip speed ratio of a WTG, dimensionless

$\lambda_i$ Intermediate variable for calculating a turbine’s power coefficient, dimensionless

$\mu_i$ $i^{th}$ Eigenvalue, $\left[ \frac{1}{s} \right]$

$\mu_i^\Re$ Real part of $i^{th}$ eigenvalue, $\left[ \frac{1}{s} \right]$

$\mu_i^\Im$ Imaginary part of $i^{th}$ eigenvalue, $\left[ \frac{\text{rad}}{s} \right]$

$\rho$ Air density, $\left[ \frac{\text{kg}}{\text{m}^3} \right]$

$\sigma$ Damping ratio of a mode, dimensionless

$\psi_{qs}, \psi_{ds}$ q-axis and d-axis stator flux-linkage, [p.u.]

$\psi_{qr}, \psi_{dr}$ q-axis and d-axis rotor flux-linkage, [p.u.]

$\omega$ Electrical angular speed of a SG, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_b$ Angular speed base, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_{\text{max}}, \omega_{\text{min}}$ WTG maximum and minimum electrical rotor speed, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_{\text{ms}}$ Machine shaft angular speed of a DFIG, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_r$ Electrical rotor speed of a DFIG, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_{\text{ref}}$ Angular speed reference of a WTG, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_s$ Synchronous angular speed of a SG, $\left[ \frac{\text{rad}}{s} \right]$

$\omega_{\text{turbine}}$ Angular speed of a wind turbine, $\left[ \frac{\text{rad}}{s} \right]$

$\eta_{\text{gen}}$ Efficiency of a DFIG, dimensionless
SYMBOLS

$A_s, B_s, C_s, D_s$ \hspace{1em} Sub-matrices of the full Jacobian matrix of a power system

$A_{sys}$ \hspace{1em} Reduced Jacobian of a power system

$A_{wt}$ \hspace{1em} Wind turbine swept area, [m$^2$]

$a, b, c$ \hspace{1em} Parameters of the reduced-order model of a wind farm’s reactive power absorption

$B$ \hspace{1em} Hybrid B-matrix of a two port network, [p.u.]

$B$ \hspace{1em} Torque parameter for calculating either mechanical torque or power of a WTG, \( \left[ \frac{\text{pu.s}^3}{\text{m}^3} \right] \)

$C$ \hspace{1em} Power parameter for calculating the power reference of a WTG, [p.u. s$^3$]

$C_p$ \hspace{1em} Power coefficient of a wind turbine, dimensionless

$D$ \hspace{1em} Turbine parameter for calculating the tip speed ratio of a WTG, dimensionless

$E'_q, E'_d$ \hspace{1em} q-axis and d-axis transient internal voltage of a SG, [p.u.]

$E'_{qD}, E'_{dD}$ \hspace{1em} q-axis and d-axis transient internal voltage of a DFIG, [p.u.]

$E_{fd}$ \hspace{1em} Field voltage of a SG, [p.u.]

$f_{osc}$ \hspace{1em} Oscillation frequency, [Hz]

$G$ \hspace{1em} Hybrid G-matrix of a two port network, [p.u.]

$H$ \hspace{1em} Inertia constant of a SG, [s]

$H_D$ \hspace{1em} Inertia constant of WTG, [s]

$I_{ar}, I_{as}$ \hspace{1em} Rotor and stator complex current of a DFIG (subindex a stands for phase-a), [p.u.]

$I_{GC}$ \hspace{1em} Complex current absorbed by the grid side converter of WTG, [p.u.]

$I_L$ \hspace{1em} Complex current absorbed by a load, [p.u.]

$I_q, I_d$ \hspace{1em} q-axis and d-axis stator currents of a SG, [p.u.]

$I_{qr}, I_{dr}$ \hspace{1em} q-axis and d-axis rotor currents of a DFIG, [p.u.]
$I_{qr}^{\text{ref}}, I_{dr}^{\text{ref}}$  
q-axis and d-axis rotor current reference of a WTG’s controllers, [p.u.]

$I_{qs}, I_{ds}$  
q-axis and d-axis stator currents of a DFIG, [p.u.]

$K_A$  
Regulator gain of the IEEE Type-1 exciter of a SG, [p.u.]

$K_E$  
Gain of the IEEE Type-1 exciter of a SG, [p.u.]

$K_F$  
Feedback gain of the IEEE Type-1 exciter of a SG, [p.u.]

$K_P, K_I$  
Proportional and integral gains of the pitch angle controller, [p.u.]

$K_{P1}, K_{I1}$  
Proportional and integral gains of the speed controller’s slow loop, [p.u.]

$K_{P2}, K_{I2}$  
Proportional and integral gains of the speed controller’s fast loop, [p.u.]

$K_{P3}, K_{I3}$  
Proportional and integral gains of the reactive power controller’s slow loop, [p.u.]

$K_{P4}, K_{I4}$  
Proportional and integral gains of the reactive power controller’s fast loop, [p.u.]

$K_{P5}, K_{I5}$  
Proportional and integral gains of a SG’s reactive power controller, [p.u.]

$k$  
Gearbox transformation ratio of a WTG, dimensionless

$P_{ag}$  
Power sent through the airgap of a DFIG, [p.u.]

$P_{\text{gen}}$  
Total active power injected by a WTG, [p.u.]

$P_l$  
Active power losses of a DFIG, [p.u.]

$P_L, Q_L$  
Active and reactive power of the load, [p.u.]

$P_m$  
Mechanical power extracted from a wind turbine, [p.u.]

$P_M$  
Mechanical power extracted from a wind turbine, [W]

$P_{\text{max}}^m$  
Maximum mechanical power extracted from a wind turbine for a given $v_{\text{wind}}$, [p.u.]

$P_{\text{max}}$  
Maximum mechanical power extracted from a wind turbine (power at $\omega_{\text{max}}$), [p.u.]

$P_r$  
Active power absorbed by the rotor of a DFIG, [p.u.]
$P_{\text{ref}}$  Active power reference of a WTG’s speed controller, [p.u.]

$P_s$  Active power produced from the stator of a DFIG, [p.u.]

$P_{sv}$  Power output of the steam valve of a SG, [p.u.] [W]

$P_{\text{wind}}(v_{\text{wind}})$  Theoretical potential power contained in an air mass at $v_{\text{wind}}$, [W]

$P_0, Q_0$  Active and reactive power of an exponential load model at a unity voltage magnitude, [p.u.]

$p$  Total pole number of a generator (SG or DFIG)

$p_{i,\mu}$  Participation factor of $i^{\text{th}}$ variable over eigenvalue $\mu$, dimensionless

$pQ$  Parameter vector of the reduced-order model of the reactive power absorption of a wind farm

$pv, qv$  Parameters of an exponential load model, dimensionless

$Q_{\text{gen}}$  Total reactive power injected by a WTG, [p.u.]

$Q_{\text{ref}}$  Reference of the reactive power controller of a SG or DFIG, [p.u.]

$Q_s$  Reactive power produced from the stator of a DFIG, [p.u.]

$R$  Turbine radius of a WTG, [m]

$R_D$  Statism constant of a SG, [p.u.]

$R_f$  Intermediate state variable of the voltage regulator of a SG, [p.u.]

$R_r, R_s$  Rotor and stator resistance of a DFIG, [p.u.]

$r, z$  Vectors of relevant and irrelevant variables

$S_b$  Base power, [MVA]

$S$  Laplace operator

$s$  Slip of a DFIG, dimensionless

$T$  Time constant of the pitch angle controller, [s]

$T_A$  Regulator time constant of the IEEE Type-1 exciter of a SG, [s]
\(T_b\) Torque base, [Nm]
\(T_{CH}\) Steam-chest time constant of the linear speed controller of a SG, [s]
\(T_e\) Electrical torque of a generator (SG or DFIG), [p.u.]
\(T_E\) Time constant of the IEEE Type-1 exciter of a SG, [s]
\(T_F\) Feedback time constant of the IEEE Type-1 exciter of a SG, [s]
\(T_m\) Mechanical torque (SG or WTG), [p.u.]
\(T_M\) Mechanical torque of a WTG, [Nm]
\(T_{q0}', T_{d0}'\) q-axis and d-axis transient time constant of a SG, [s]
\(T_{SV}\) Steam-valve time constant of the linear speed controller of a SG, [s]
\(T_0'\) Transient open-circuit time constant of a DFIG, [s]
\(\bar{V}_{ar}, \bar{V}_{as}\) Rotor and stator complex voltage of a DFIG (subindex a stands for phase-a), [p.u.]
\(V_b\) Base voltage, [kV]
\(V_D\) Terminal voltage magnitude of a DFIG, [p.u.]
\(V_L\) Terminal voltage magnitude of a load, [p.u.]
\(V_S\) Terminal voltage magnitude of a SG, [p.u.]
\(V_{qr}, V_{dr}\) q-axis and d-axis rotor voltages of a DFIG, [p.u.]
\(V_{qs}, V_{ds}\) q-axis and d-axis stator voltages of a DFIG, [p.u.]
\(V_{ref}\) Voltage controller reference of a SG or WTG, [p.u.]
\(V_R\) Voltage regulator output of a SG, [p.u.]
\(v_{tip}\) Speed of a blade tip in a WTG, \([\frac{m}{s}]\)
\(v_{wind}\) Wind speed, \([\frac{m}{s}]\)
\(v\) Right eigenvector
\(w\) Left eigenvector
\(X_{tr}, X_{ts}\) Rotor and stator leakage reactance of a DFIG, [p.u.]
\( X_m \)  Magnetizing reactance of a DFIG, [p.u.]
\( X_q, X_d \)  q-axis and d-axis steady-state reactance of a SG, [p.u.]
\( X'_q, X'_d \)  Transient q-axis and d-axis reactance of a SG, [p.u.]
\( X_r, X_s \)  Rotor and stator reactance of a DFIG, [p.u.]
\( X'_s \)  Transient reactance of a DFIG, [p.u.]
\( x_1, x_2 \)  State variables related to the speed controller of a WTG, [p.u.]
\( x_3, x_4 \)  State variables related to the reactive power controller of a WTG, [p.u.]
\( x_5, x_6 \)  State variables related to the pitch angle controller of a WTG, [p.u.]
\( Y \)  Admittance matrix of a two port network, [p.u.]
\( y M \)  Vector and matrix of measurements of the least squares method
\( Z \)  Impedance matrix of a two port network, [p.u.]
CHAPTER 1
INTRODUCTION

Worldwide energy consumption has steadily increased in recent decades due to the rate of growth in world *gross domestic product*—the main driver of energy demand. It is expected that the electricity demand will increase at a rate of 2.6% per year during the period 2004-2030. Additionally, global energy-related carbon-dioxide emissions, a major cause of global warming, are expected to increase by 1.7% per year during the same period—reaching $40.4 \times 10^9$ tons in 2030. Unfortunately, power generation is projected to contribute almost 50% of that increased emission [1]. Thus, the power generation sector is under scrutiny; it has to be expanded in order to fulfill the high-energy-demand scenario while also taking into account environmental effects such as global warming.

1.1 Power System Expansion and Operation

Proper expansion of a power system must take into account economic and technical considerations such as *minimum operating cost*, *system reliability* and *system security*. *Minimum operating cost* is obtained by scheduling the operation of generating units based on their cost, response speed and system loading cycle. The well-known problems of economic dispatch and unit commitment are solved for that purpose. *System reliability* has to do with the system’s ability to serve the load at any instant. Power reserve and network configuration as well as system contingency analysis are issues of extreme importance for having a reliable system. *System security* is related to the system’s ability to keep its variables within acceptable bandwidths not harmful for system elements, e.g., generators, transformers, loads, etc. System security is typically defined using three operating states: normal, emergency and restorative states [2]. Any transition between these states will evolve
dynamically. The existence of stable equilibrium points is mandatory to operate the system safely. Proper dynamic models are required. These models are mainly focused on low frequency oscillations between 0.2 and 2 [Hz] [3]. Specifically, this research deals with obtaining a proper model for WTGs that can be used in power system stability analysis. Note that dynamics associated with flux linkages in machines’ stators and transmission lines are typically assumed to be infinitely fast. Therefore, they are modeled using algebraic equations which allow them to change instantaneously depending on the system state.

1.2 Wind Power Generation

Renewable energy sources are needed to face the near future energy scenario. Hydraulic, wind, solar, biomass and geothermal sources are the most common renewable generation systems. Hydraulic systems are attractive due to their robustness, reliability and high rated power levels. However, the main drawbacks are the scarce available locations with hydro potential and the negative impact on the local ecosystem by flooding extensive areas. At present, among the other alternatives, wind generation systems are the most qualified to produce electricity. Although being irregular in their electricity production, wind farms are able to provide energy: (a) without the risk of depletion of their primary energy source, and (b) in compliance with operation standards. Additionally, wind generation systems have the fastest payback period [4], less than a year; the lowest installation period, due to their modularity; and low operation-maintenance cost [5]. These characteristics make wind power attractive for mass production, reflected in the increase of the worldwide installed power capacity in recent years [6].

Four types of wind power generation systems are known in the literature. Type-A and Type-B configurations are based on induction generators. While Type-A provides a fixed-speed operation by just using stall and/or pitch control, Type-B provides a limited variable-speed operation by additionally controlling the rotor circuit resistance. The main disadvantages of these configurations are the power fluctuations transmitted to the network due to wind speed variations [7]. Type-C configuration is based on doubly-fed induction generators providing a variable-speed operation. Power output
is controlled over a wider range by changing the voltage applied to the rotor circuit. Finally, Type-D configuration is based on a full-scale ac/ac converter which decouples the generator and system frequency. It uses either induction or synchronous generators. This configuration does not require a gearbox, increasing its reliability [8]. The main advantages of Type-C and Type-D configurations are the reduction of power output fluctuations and optimal power extraction from the wind. Type-C configuration is used in this proposed research as it is the most common in current wind farm projects.

1.3 Wind Power Generation in Power Systems

A power system can be defined as a system that generates electrical power at large scale which is then sent to consumers by means of electrical lines. As the energy sources are generally located far from the demand centers, the voltage of electrical lines is increased to reduce power losses. Thus, electrical transformers are required. Based on the different voltage levels, different power system sectors are defined. In addition, based on the purpose of every sector, the generation (voltage levels of about 12 kV), transmission (voltage levels of about 220-500 kV], sub-transmission (voltage levels of about 66-110 kV) and distribution areas (voltage levels of about 12-15 kV) are defined. The voltage level and power capability of the areas are reduced when they are close to the final clients. Note that, typically, small wind farms are located in the distribution area while larger wind farms are located in the sub-transmission area.

Power system generators and loads are coupled through the grid, which is the generic name for all the elements that belong to the transmission, sub-transmission and distribution areas. By assuming that the flux linkage dynamics of the grid’s elements are infinitely fast, they are modeled by algebraic equations. It turns out that the grid is represented by using Kirchoff’s law and the grid’s steady-state parameters of series resistance, series reactance and parallel admittance. Thus, the grid model is $0 = g(V, \theta, P, Q)$, where $V$ and $\theta$ are the vectors of voltage magnitude and voltage angle; $P$ and $Q$ are the vectors of active power and reactive power injections; $g, V, \theta, P, Q \in \mathbb{R}^{2n \times 1}$, where $n$ is the total number of buses; and $g$ is nonlinear.

Synchronous generators (SGs) are used in the generation area. The key
characteristic of a SG is that the rotational frequency of its shaft is proportional to the frequency of its stator voltage and current. Thus, by adjusting the mechanical torque applied to the shaft, the electrical frequency can be controlled. In Figure 1.1, a power system scheme is presented. Due to the grid coupling, the stator current and voltages in all SGs must have the same frequency. The equations of motion of the SGs and the equations of the frequency control are presented. Note that SGs are coupled to each other by the frequency control and by the grid. The frequency control is performed in two phases. The first one is called the primary frequency control in which the SG’s mechanical power is adjusted based on frequency variations. For example, if $\omega_1 > \omega_s$ then the mechanical power is reduced depending on the statism constant $R_{D1}$. The second one is called the secondary frequency control and has to do with the adjustment of the mechanical power set-points, e.g., $P_{\text{set-point}}$. On the other hand, Type-C WTGs are based on doubly-fed induction generators which are asynchronous machines. The rotational frequency of their shaft is not related to the frequency of their terminal voltages and currents. In fact, the rotor angular speed of a WTG just participates in the energy conversion, and it can be established that a WTG’s power output is a function of the WTG’s angular speed and wind speed. Thus, the only coupling of a WTG with other generators is through the grid. Based on this characteristic, it seems that a complex dynamic model for a WTG is not required in power system analysis. In fact, several publications have pointed out that WTGs do not have a major impact on power system dynamics [9–12]. The most influential component of a WTG model is the voltage controller, which affects oscillations damping [13, 14]. A simplified WTG model is desired as the high number of turbines in a wind farm makes the system more complex, and a detailed model can place an unsustainable burden on a simulation program. Another important issue is the energy storage. While SGs can regulate their power output by using their energy storage, e.g., coil or hydro reservoir, WTGs produce power from the available wind. As the wind is intermittent, so is the power output. Therefore, in order to balance demand and generation, SGs have to adjust their power output due to a variable wind power production. Any deficit or excess of power from the WTGs will be compensated by the SGs. This is the major interaction between WTGs and SGs. A simplified WTG model based on power injection is proposed in this thesis. WTGs with frequency response, which can inject
more or less power based on frequency deviation, are beyond the scope of this thesis.

1.4 Wind Farm Model for Power System Stability Analysis

In this thesis, the modeling of wind farms based on Type-C WTGs is studied. Based on time scale decomposition, two detailed dynamic models are presented. In the first model, while rotor flux linkage dynamics are fully modeled, stator flux linkage dynamics are assumed to be infinitely fast. This representation is called the two-axis model. In the second model, both rotor and stator flux linkage dynamics are assumed to be infinitely fast. This is called the zero-axis model. In both models, a rotor speed controller, a reactive power controller and a pitch angle controller are considered. The turbine’s aerodynamic is represented by a static model which relates the wind speed, the rotor speed and the mechanical power extracted from the wind. The gearbox is assumed to be stiff and, therefore, a single-mass model is assumed. This single-mass model basically considers the turbine, the gear-
box and generator’s shaft as a whole and represents them by a unique inertia constant. With respect to the controllers, the speed controller is designed to extract maximum power from the wind for a given wind speed. The reactive power controller is designed to follow a reference which is, in general, a zero reactive power output. Both speed and reactive power controllers use proportional and integral control with an internal and external control loop. The pitch angle controller is designed to limit the active power output and it also considers a proportional and integral control. While the two-axis model contains 10 differential and 8 algebraic equations, the zero-axis model contains 8 differential and 8 algebraic equations. In most of the analysis, the wind speed, and therefore the active power output, is assumed to be within its limits. Consequently, the pitch angle controller is not taken into account. The numbers of differential equations of the two-axis and zero-axis models are reduced to 7 and 5, respectively. When every single WTG in a wind farm is represented by these models, the power system model’s dimensionality increases notably.

Modal and bifurcation analysis reveals that typically a WTG’s variables participate in those modes that lie close to the real axis of the complex plane. Thus, a WTG’s variables do not exhibit major oscillatory behavior when the system is perturbed. This phenomenon is observed during time domain simulations. In addition, WTG’s variables, most of the time, do not participate in unstable modes. In power systems, there is generally a complex pair of eigenvalues that crosses the imaginary axis when the load is increased. This complex pair of eigenvalues defines the system Hopf bifurcation (HB) point and is related to variables of SGs’ voltage controller. This is observed with and without a WTG. Thus, a WTG does not have a major impact on the system stability and the most important interaction with the system is the interchange of power. This supports the fact that a negative load model can represent WTGs in power system analysis.

In order to obtain a wind farm’s simplified model, both model aggregation and model order reduction are considered. Basically, the aggregation technique used in this thesis gives a single equivalent WTG by aggregating the mechanical power extracted from the turbines. In other words, an equivalent mechanical power is obtained by summing over all WTGs’ mechanical power. As a result, an equivalent speed is defined. In addition, equivalent parameters and variables are defined using the same concept of adding powers. The
equivalent WTG has the same characteristics as individual WTGs and also has equivalent controllers. When the equivalent WTG’s model is obtained, the model order is reduced by using selective modal analysis. When static and dynamic analysis are performed, results prove that a negative load model for representing WTGs in a power system is possible. Replacing either a two-axis or zero-axis model of a WTG by a negative load neither considerably alters the original system’s dynamics nor modifies the system’s variables. This is beneficial due to the reduction of simulation time and model complexity. In the largest case, it is shown that two wind farms that in total are represented by 500 differential equations and 800 algebraic equations can be represented by just 4 differential equations. There are other factors that should be included in this study, such as equivalency during short-circuits, different control schemes, wind farm loss estimation, and wind farm central control, among others. All these issues are left for future research.

This thesis is structured into three main chapters. In Chapter 2, modeling issues of a WTG for steady-state and dynamic analysis are presented. The two-axis and zero-axis models are fully described. Based on these models, the power capability of a WTG is studied. The ability of a Type-C WTG to provide reactive power to the grid is analyzed. In addition, discussions about reactive power control versus voltage control and about speed controller sub-optimality are presented. Finally, modal analysis is performed to a 4-bus test system with 1 SG and 1 WTG. Finally, a reduced-order model of the WTG is obtained. In Chapter 3, the aggregated model is presented which is compared to Slootweg's method. In Chapter 4, the aggregation and reduction of the model order are applied to the New England Test System, a 39-bus, 9-machine system. Three wind power scenarios are considered. In the largest case, two wind farms of 50 WTGs are considered. The system’s full order model, the aggregated model and reduced-order model are compared for several wind speed profiles and also when some lines and generators go out of service.
CHAPTER 2

WIND POWER GENERATION

In 1920, Albert Betz, a German pioneer of wind power technology, studied the best utilization of wind energy in windmills, establishing a theoretical limit of power extraction. Basically, the theory said that, regardless of the turbine design, at most $\frac{16}{27} \times 100\% \approx 59.3\%$ of the wind kinetic energy can be converted into mechanical energy [8].

In order to understand power extraction from wind, it is required to define the tip speed ratio, $\lambda$, which is the ratio between the speed of a blade tip, $v_{tip}$ [m/s], and the wind speed, $v_{wind}$ [m/s]. Thus, $\lambda = \frac{v_{tip}}{v_{wind}} = \frac{\omega_{turbine} R}{v_{wind}}$, where $R$ is the turbine radius. Then, the mechanical power extracted from a wind turbine can be estimated by [8]

$$P_M = C_p(\lambda, \theta) P_{wind}(v_{wind}) = C_p(\lambda, \theta) \frac{1}{2} \rho A_{wt} v_{wind}^3 \text{ [W]} \quad (2.1)$$

where $\rho$ is the air density [kg/m$^3$], $A_{wt} = \pi R^2$ is the wind turbine swept area [m$^2$] and $v_{wind}$ is the wind speed [m/s]. $P_{wind}(v_{wind}) = \frac{1}{2} \rho A_{wt} v_{wind}^3$ is the theoretical potential power contained in an air mass at $v_{wind}$. $C_p$ is the power coefficient, which is dimensionless and depends on both the tip speed ratio, $\lambda$, and the pitch angle, $\theta$ [degrees]—angle of incidence of a turbine’s blade and the wind direction. This coefficient takes into account the turbine’s aerodynamic and establishes the fraction of the potential power that can be extracted. Note that $C_p$ is less than Betz’s limit, i.e., $\forall \lambda, \theta \quad C_p(\lambda, \theta) < 0.593$.

Using a fixed pitch angle, typical power curves as a function of the wind speed and the turbine angular speed are depicted in Figure 2.1. Note that at every wind speed there is an optimum turbine speed at which the power extraction from the wind is maximized.

In the 1990s, typical wind power turbines were characterized by a fixed-speed operation. They were based on induction generators directly connected to the grid. Additionally, a soft starter to energize the machine and a bank
of capacitors to compensate reactive-power absorption were used. Although simple and reliable, the fixed-speed wind turbines were inefficient, and power fluctuations were transmitted to the network due to wind speed variations [7].

In the mid-1990s, variable-speed wind power turbines gave a boost to the wind power industry. A reduction in the power fluctuations and an optimal power extraction from the wind were possible by the development of new turbine control systems. Among the different configurations, the Type-C based on a doubly-fed induction generator (DFIG), at present, is the most used in the development of new wind farm projects. This configuration consists of the coupling of a turbine, a gearbox and an induction generator doubly connected to the grid—directly connected from the stator circuit and indirectly connected from the rotor circuit by using converters. Its main drawbacks are the use of slip rings and protection in case of grid disturbances [8]. The control can be done (a) by controlling the voltage applied to the rotor circuit, (b) by adjusting the pitch angle, and (c) by designing the turbine blades to stall when the wind speed exceeds its limit [15].

The basic components of Type-C WTGs are the turbine, gearbox, doubly-fed induction generator, grid-side converter, dc-link and rotor-side converter (see Figure 2.2). Gearbox allows increasing the angular speed from the turbine to the generator shaft. The converters and dc-link feed the low-frequency rotor circuit from the grid and are partially scaled, requiring a rated power of about 30% of the generator rating. Usually, the generator slip varies between 40% at sub-synchronous speed and −30% at super-synchronous speed [8]. With respect to the control, the grid-side converter is typically controlled to have a unity power factor and a constant voltage at the dc-link. The rotor-
side converter is usually controlled to have (a) optimal power extraction from the wind and (b) a specified reactive power at the generator terminal. Why reactive power control is considered instead of voltage control will be discussed in Section 2.6. Note that the rotor-side converter provides sinusoidal three-phase voltages at the slip frequency. Therefore, assuming that the converters and dc-link are lossless, the net power injected by the generator to the grid is

\[ P_{\text{gen}} = P_s - P_r \quad Q_{\text{gen}} = Q_s \quad (2.2) \]

where \( P_s \) and \( Q_s \) are the active and reactive power going out of the stator. \( P_r \) is the active power injected by the rotor-side converter to the rotor circuit (it is also equal to the active power absorbed by the grid-side converter from the grid).

Due to the importance of wind power generation on the current and future worldwide energetic scenario, dynamic models are required for research, teaching and training purposes. One major problem is that dynamic models of commercial wind power turbines contain proprietary information and require confidentiality agreements between the company and the user [16]. In the literature, there are several articles dealing with different issues related to WTGs [8, 17–27], but only a general description of the dynamic model is found. From the literature, a generic dynamic model of a Type-C WTG...
including active and reactive power controllers is derived and presented in
the next section.

2.1 Dynamic Model

A general scheme of the Type-C WTG is shown in Figure 2.3. The turbine
is modeled by Equation (2.1) and using an intermediate parameter, \( \lambda_i \), the
power coefficient is estimated by [24]

\[
C_p(\lambda_i, \theta) = 0.22 \left( \frac{116}{\lambda_i} - 0.4\theta - 5 \right) e^{-\frac{12.5}{\lambda_i}} \tag{2.3}
\]

where \( \lambda_i = \left( \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{\theta^3 + 1} \right)^{-1} \) \hspace{1cm} (2.4)

The gearbox model depends on its stiffness. If the gearbox has some degree
of flexibility, a two-mass model is typically used which separately considers
the mass of the turbine and the low speed side of the gearbox, and the mass
of the generator and the high speed side of the gearbox. In this research,
the gearbox is assumed to be stiff and the masses of the turbine, gearbox
and generator are considered as a whole (single mass model). In addition, a
constant speed transformation ratio, \( k \), is considered. A relationship between
the turbine angular speed, \( \omega_{turbine} \), the machine shaft angular speed, \( \omega_{ms} \), and
the electrical rotor speed, \( \omega_r \), is established as \( \omega_{turbine} = k\omega_{ms} = \frac{2k}{p}\omega_r \). Thus,
the tip speed ratio as a function of \( \omega_r \) and \( v_{wind} \) is

\[
\lambda = \frac{v_{tip}}{v_{wind}} = \frac{2k}{p} \frac{\omega_r R}{v_{wind}} \tag{2.5}
\]

where \( p \) is the generator’s total number of poles. Rotor-side and grid-side
converters are modeled as a controlled voltage source and a controlled current
source, respectively. Control systems for pitch angle, rotor speed (or active
power) and reactive power are typically modeled using PI-controllers [25–27]
(see Figures 2.4, 2.5 and 2.6). Active and reactive power control schemes
consider both an internal current loop (fast response) and an external power
loop (slow response). Active power is tracked for optimal power extraction
from the wind. Reactive power control uses a fixed set-point.

The induction machine model requires the specification of a reference
frame. A stationary reference frame, a rotor reference frame or a synchronously rotating reference frame can be used [28]. The last is adopted in this article. Note that a generator convention is used; i.e., the stator and rotor currents are positive when they are leaving and entering the machine, respectively. Basically, a model using $a$-$b$-$c$ stator- and rotor-phases is referred to a particular reference frame with two orthogonal axes, the quadrature axis ($q$-axis) and the direct axis ($d$-axis). Note that the $q$-axis leads the $d$-axis by 90 degrees. Also, a zero-sequence component which is arithmetically related to the $a$-$b$-$c$ variables is defined. This zero component takes into account any imbalance of the original three-phase variables. The transfor-
mation matrix, \( T \), between the variables in the \( a-b-c \) to the \( q-d-0 \) reference is defined as

\[
\begin{bmatrix}
  f_q \\
  f_d \\
  f_0
\end{bmatrix}
= T
\begin{bmatrix}
  f_a \\
  f_b \\
  f_c
\end{bmatrix}
\]  \hspace{1cm} (2.6)

\[
T = \frac{2}{3}
\begin{bmatrix}
  \cos \theta_{\text{frame}} & \cos(\theta_{\text{frame}} - \frac{2\pi}{3}) & \cos(\theta_{\text{frame}} + \frac{2\pi}{3}) \\
  \sin \theta_{\text{frame}} & \sin(\theta_{\text{frame}} - \frac{2\pi}{3}) & \sin(\theta_{\text{frame}} + \frac{2\pi}{3}) \\
  \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]  \hspace{1cm} (2.7)

where \( f_a, f_b \) and \( f_c \) are arbitrary variables in phases \( a-b-c \) and \( f_q, f_d \) and \( f_0 \) are their corresponding transformed variables in the \( q- \) and \( d- \) axis and zero component, respectively. Also, \( \theta_{\text{frame}} \) is the angle between the voltage vector \( f_a \) and the \( q- \) axis as shown in Figure 2.7.

A torque expression is required to model the motion of the rotatory mass—turbine, gearbox and machine shaft. Assume that the power transmission from the wind turbine to the machine shaft is lossless. Then, the torque in \([\text{Nm}]\) at the machine shaft is

\[
T_M = \frac{P_M}{\omega_{\text{ms}}} = \frac{k}{2\lambda} \rho \pi R^3 C_p(\lambda, \theta) v_{\text{wind}}^2 \text{ [Nm]} \]  \hspace{1cm} (2.8)

Use the per-unit base of the induction-machine to define the power base, \( S_b \), the voltage base, \( V_b \), and the electrical speed base, \( \omega_b \). The torque base can be calculated as \( T_b = S_b \frac{P}{2\omega_b} \). More details about the per unit system in electrical machines can be found in [3, 28, 29]. As in the next sections all

![Figure 2.7: Synchronously rotating reference frame.](image)
expressions are in per unit, the torque equation is scaled as

$$T_m = \frac{T_M}{T_b} = \frac{1}{2} \frac{\rho \pi R^2 \omega_b}{S_b \omega_r} C_p(\lambda, \theta) v_{\text{wind}}^3 \text{ [p.u.]}$$

$$= B \omega_b C_p(\lambda, \theta) \frac{v_{\text{wind}}^3}{\omega_r} \text{ [p.u.]} \quad (2.9)$$

where $B = \frac{1}{2} \frac{\rho \pi R^2}{S_b}$. Finally, depending on the flux linkage dynamics of the generator itself, two models of different order are presented. For the sake of clarity, consider a Type-C WTG connected to an infinite bus through an electrical line of short length; i.e., just its series resistance and reactance are considered.

### 2.1.1 Two-Axis Model (10th Order Model)

By analogy to the two-axis model for synchronous generators, this Type-C WTG model is called the two-axis model because while it just represents the dynamics of the rotor flux linkages in the $q$-axis and $d$-axis, the dynamics of stator flux linkages are assumed to be infinitely fast. For now, let us just consider the model of the electrical generator itself. A DFIG is simply a wound rotor induction machine where the stator- and rotor-circuits are energized. Both stator and rotor windings participate in the electromechanical energy conversion. Consequently, a DFIG model is basically the same as the model of an induction machine in which the rotor voltages are supplied by an electric source, i.e., rotor side converter. This configuration allows the machine to operate in a wide speed range. Therefore, the model of the stator and rotor of the DFIG in terms of their flux linkages is the following:

\[
\begin{align*}
\frac{1}{\omega_s} \frac{d\psi_{qs}}{dt} &= V_{qs} + R_s I_{qs} - \psi_{ds} \quad (2.10) \\
\frac{1}{\omega_s} \frac{d\psi_{ds}}{dt} &= V_{ds} + R_s I_{ds} + \psi_{qs} \quad (2.11) \\
\frac{1}{\omega_s} \frac{d\psi_{qr}}{dt} &= V_{qr} - R_r I_{qr} - \frac{(\omega_s - \omega_r)}{\omega_s} \psi_{dr} \quad (2.12) \\
\frac{1}{\omega_s} \frac{d\psi_{dr}}{dt} &= V_{dr} - R_r I_{dr} + \frac{(\omega_s - \omega_r)}{\omega_s} \psi_{qr} \quad (2.13)
\end{align*}
\]
\[ \psi_{qs} = -X_s I_{qs} + X_m I_{qr} \quad (2.14) \]
\[ \psi_{ds} = -X_s I_{ds} + X_m I_{dr} \quad (2.15) \]
\[ \psi_{qr} = -X_m I_{qs} + X_r I_{qr} \quad (2.16) \]
\[ \psi_{dr} = -X_m I_{ds} + X_r I_{dr} \quad (2.17) \]

where \( V, I, R, X \) and \( \psi \) correspond to the voltages [p.u.], currents [p.u.], resistances [p.u.], reactances [p.u.] and flux linkages [p.u.], respectively. Also, \( X_m \) is the mutual reactance between the stator and the rotor, \( X_s = X_{\ell s} + X_m \) is the stator reactance and \( X_r = X_{\ell r} + X_m \) is the rotor reactance. \( X_{\ell s} \) and \( X_{\ell r} \) are the stator and rotor leakage-reactance, respectively. All variables and parameters are in per unit except \( \omega_r \) and \( \omega_s \).

In dynamics simulations, it has been observed that stator dynamics are faster than rotor dynamics [28,30,31]. In order to visualize this phenomenon, solve for \( I_{dr} \) and \( I_{qr} \) from Equations (2.16)-(2.17):

\[ I_{qr} = \frac{X_m}{X_r} I_{qs} + \frac{\psi_{qr}}{X_r} \]
\[ I_{dr} = \frac{X_m}{X_r} I_{ds} + \frac{\psi_{dr}}{X_r} \quad (2.18) \]

Multiply Equations (2.12)–(2.13) by \( \frac{X_m}{R_r} \) and replace \( I_{qr} \) and \( I_{dr} \) to obtain

\[ \frac{X_r}{\omega_s R_r} \frac{d}{dt} \left( \frac{X_m}{X_r} \psi_{dr} \right) = \frac{X_m}{R_r} V_{dr} - \frac{X_m}{X_r} \psi_{dr} - \frac{X_m^2}{X_r} I_{ds} \\
\quad + \frac{(\omega_s - \omega_r)}{\omega_s} X_r \left( \frac{X_m}{X_r} \psi_{qr} \right) \quad (2.19) \]
\[ \frac{X_r}{\omega_s R_r} \frac{d}{dt} \left( \frac{X_m}{X_r} \psi_{qr} \right) = \frac{X_m}{R_r} V_{qr} - \frac{X_m}{X_r} \psi_{qr} - \frac{X_m^2}{X_r} I_{qs} \\
\quad - \frac{(\omega_s - \omega_r)}{\omega_s} X_r \left( \frac{X_m}{X_r} \psi_{dr} \right) \quad (2.20) \]

Define

\[ T'_0 = \frac{X_r}{\omega_s R_r} \quad X'_s = X_s - \frac{X_m^2}{X_r} \quad (2.21) \]
\[ E'_{qD} = \frac{X_m}{X_r} \psi_{dr} \quad E'_{dD} = -\frac{X_m}{X_r} \psi_{qr} \quad (2.22) \]

where \( T'_0 \) is the transient open-circuit time constant, \( X'_s \) is the transient reactance, \( E'_{qD} \) and \( E'_{dD} \) are the \( q \)- and \( d \)-axis transient rotor-voltages, respectively. Note that the sub-index \( D \) stands for DFIG. For large machines
Thus, stator dynamics are faster than rotor dynamics [30,31]. Using a zero-order integral manifold to represent the stator dynamics, the reduced-order model of the generator itself becomes

\begin{align*}
V_{qs} &= -R_s I_{qs} - X'_s I_ds + E'_{qD} \\
V_{ds} &= -R_s I_{ds} + X'_s I_{qs} + E'_{dD} \\
T_0 \frac{dE'_{qD}}{dt} &= - (E'_{qD} + (X_s - X'_s) I_{ds}) \\
&+ T_0' \left( \omega_s \frac{X_m}{X_r} V_{dr} - (\omega_s - \omega_r) E'_{dD} \right) \\
T_0 \frac{dE'_{dD}}{dt} &= - (E'_{dD} - (X_s - X'_s) I_{qs}) \\
&+ T_0' \left( -\omega_s \frac{X_m}{X_r} V_{qr} + (\omega_s - \omega_r) E'_{qD} \right)
\end{align*}

To obtain the stator algebraic Equations (2.23)–(2.24), use Equations (2.14)-(2.15), (2.18) and (2.22). Rotor flux equations can be stated in terms of the new variables $E'_{qD}$ and $E'_{dD}$ as

\begin{align*}
I_{dr} &= \frac{E'_{qD}}{X_r} + \frac{X_m}{X_r} I_{ds} \\
I_{qr} &= - \frac{E'_{dD}}{X_m} + \frac{X_m}{X_r} I_{qs}
\end{align*}

Note that the relation between the variables in the synchronously rotating reference frame and the variables in phasor representation is given by

\begin{equation}
\mathbf{F} = F_q - jF_d
\end{equation}

where $F$ is any arbitrary variable [28]. Thus, multiplying Equation (2.24) by $e^{-j\frac{\pi}{2}}$ and adding Equation (2.23), a phasor machine representation to calculate stator algebraic-variables, given $E'_{qD}$ and $E'_{dD}$, is obtained.

\begin{equation}
V_D e^{j\theta_D} = E'_{qD} - j E'_{dD} = (R_s + j X'_s)(I_{qs} - j I_{ds}) + V_{qs} - jV_{ds}
\end{equation}

$V_D$ and $\theta_D$ are the magnitude and angle of the complex voltage at the stator terminal of the DFIG. The relationship between the phasor voltage and the $q$-axis and $d$-axis voltages can be found in [28].

Speed and reactive power controllers are designed based on the decoupling of active- and reactive-power output. If the $d$-axis is oriented along the stator
flux axis, then $V_{qs} = V_D$, $V_{ds} = 0$ and

$$P_s = \frac{X_m}{X_s} V_s I_{qr} \quad Q_s = V_s \frac{X_m I_{dr} - V_D}{X_s} \quad (2.30)$$

The alignment between these axes is obtained by field-oriented control and it implies the existence of a shift angle between the machine reference and network reference. Thus, an ideal shift-angle transformer which keeps a zero angle at the DFIG’s terminal is considered. See Appendix A for more details about the power decoupling and the alignment of these axes. To obtain a differential-algebraic model for the controllers, some additional state variables have to be defined. For example, consider the active power controller. The following algebraic relationships in the Laplace domain are obtained by inspection from its block diagram (Figure 2.4):

$$K_{P1} (P_{ref} - P_{gen}) + K_{I1} \frac{(P_{ref} - P_{gen})}{S} = I_{qr}^{ref} \quad (2.31)$$

$$K_{P2} (I_{qr}^{ref} - I_{qr}) + K_{I2} \frac{(I_{qr}^{ref} - I_{qr})}{S} = V_{qr} \quad (2.32)$$

Thus, in the time domain, the following equations are obtained:

$$\frac{dx_1}{dt} = K_{I1} (P_{ref} - P_{gen}) \quad (2.33)$$

$$\frac{dx_2}{dt} = K_{I2} (K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr}) \quad (2.34)$$

$$0 = -V_{qr} + x_2 + K_{P2} (K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr}) \quad (2.35)$$

Proceeding in the same fashion, new state variables $x_3$, $x_4$, $x_5$ and $x_6$ are defined for the reactive power and pitch angle control. With respect to the equation of motion, also known as the swing equation, a lumped model is considered in which the total inertia of the turbine, gearbox and machine shaft is represented as a whole. The electrical torque in term of the new variables is obtained as follows:
\[ T_e = \psi_{dr} I_{qr} - \psi_{qr} I_{dr} \]
\[ = \psi_{dr} \left( \frac{X_m}{X_r} I_{qs} + \frac{\psi_{qr}}{X_r} \right) - \psi_{qr} \left( \frac{X_m}{X_r} I_{ds} + \frac{\psi_{dr}}{X_r} \right) \]
\[ = \left( \frac{X_m}{X_r} \psi_{dr} \right) I_{qs} + \left( -\frac{X_m}{X_r} \psi_{qr} \right) I_{ds} + \psi_{dr} \psi_{qr} \frac{\psi_{dr}}{X_r} \psi_{dr} \]
\[ = E'_{qD} I_{qs} + E'_{dD} I_{ds} \quad (2.36) \]

Finally, assuming that the machine is connected to an infinite bus through a transmission line with a series impedance \( R + jX \), the two-axis model for the Type-C WTG is defined by the following set of differential-algebraic equations (DAEs):

**Differential Equations**

\[
\frac{dE'_{qD}}{dt} = -\frac{1}{T_0} \left( E'_{qD} + (X_s - X'_s) I_{ds} \right) + \left( \omega_s \frac{X_m}{X_r} V_{dr} - (\omega_s - \omega_r) E'_{dD} \right) \quad (2.37)
\]

\[
\frac{dE'_{dD}}{dt} = -\frac{1}{T_0} \left( E'_{dD} - (X_s - X'_s) I_{qs} \right) - \left( \omega_s \frac{X_m}{X_r} V_{qr} - (\omega_s - \omega_r) E'_{qD} \right) \quad (2.38)
\]

\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{2H_D} \left[ T_m - E'_{dD} I_{ds} - E'_{qD} I_{qs} \right] \quad (2.39)
\]

\[
\frac{dx_1}{dt} = K_{I1} \left[ P_{ref} - P_{gen} \right] \quad (2.40)
\]

\[
\frac{dx_2}{dt} = K_{I2} \left[ K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr} \right] \quad (2.41)
\]

\[
\frac{dx_3}{dt} = K_{I3} \left[ Q_{ref} - Q_{gen} \right] \quad (2.42)
\]

\[
\frac{dx_4}{dt} = K_{I4} \left[ K_{P3} (Q_{ref} - Q_{gen}) + x_3 - I_{dr} \right] \quad (2.43)
\]

\[
\frac{dx_5}{dt} = K_I (\omega_r - \omega_{ref}) \quad (2.44)
\]

\[
\frac{dx_6}{dt} = x_5 - x_6 - \theta + K_P (\omega_r - \omega_{ref}) \quad (2.45)
\]

\[
\frac{d\theta}{dt} = x_6 \quad (2.46)
\]
Algebraic Equations

\[ 0 = -V_{qr} + K_{P2} [K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr}] + x_2 \]  
\[ 0 = -V_{dr} + K_{P4} [K_{P3} (Q_{ref} - Q_{gen}) + x_3 - I_{dr}] + x_4 \]  
\[ 0 = -P_{gen} + E_{dD}' I_{ds} + E_{qD}' I_{qs} - R_s (I_{ds}^2 + I_{qs}^2) - (V_{qr} I_{qr} + V_{dr} I_{dr}) \]  
\[ 0 = -Q_{gen} + E_{qD}' I_{ds} - E_{dD}' I_{qs} - X_s' (I_{ds}^2 + I_{qs}^2) \]  
\[ 0 = -I_{dr} + \frac{E_{qD}'}{X_m} + \frac{X_m I_{ds}}{X_r} \]  
\[ 0 = -I_{qr} - \frac{E_{dD}'}{X_m} + \frac{X_m I_{qs}}{X_r} \]  

Network Algebraic Equations

\[ E_{qD}' - j E_{dD}' = (R_s + j X_s')(I_{qs} - j I_{ds}) + V_D \]  
\[ V_D e^{j \theta_D} = (R + j X)(I_{qs} - j I_{ds} - I_{GC}) e^{j \theta_D} + V e^{j \theta} \]

where

\[ \tilde{I}_{GC} = \frac{P_r}{(V_{qs} - j V_{ds})^*} = \frac{V_{qr} I_{qr} + V_{dr} I_{dr}}{V_D} \]  

Technical Limits

\[ \theta_{min} - K_P (\omega_r - \omega_{ref}) \leq x_5 \leq \theta_{max} - K_P (\omega_r - \omega_{ref}) \]  

References

\[ P_{ref} = \begin{cases} 
C \omega_r^3 & \text{if } \omega_r \leq \omega_{max} \\
max & \text{if } \omega_r > \omega_{max} 
\end{cases} \]  
\[ Q_{ref} = Q_{set-point} \text{ a fixed reference} \]  
\[ \omega_{ref} = \frac{P_{ref}}{T_m} \]

Note that the algebraic equations and network algebraic equations can be alternatively represented by the equivalent circuit of Figure 2.8, where \( \tilde{I}_{GC} \) is just expressed in terms of the state variables \( x_1, x_2, x_3 \) and \( x_4 \) and the algebraic variables \( I_{qs}, I_{ds}, V_D \) and \( \theta_D \) which belong to the equivalent circuit.
Then,

\[ I_{GC} = \frac{K_{P_2}}{V_D} \left\{ -\frac{E_{dD}'}{X_m} + \frac{X_m}{X_r} I_{qs} \right\} \]

\[ + \left\{ \frac{x_2}{K_{P_2}} + K_{P_1}(P_{ref} - P_{gen}) + x_1 + \frac{E_{dD}'}{X_m} - \frac{X_m}{X_r} I_{qs} \right\} \]

\[ + \frac{K_{P_4}}{V_D} \left\{ \frac{E_{qD}'}{X_m} + \frac{X_m}{X_r} I_{ds} \right\} \]

\[ + \left\{ \frac{x_4}{K_{P_4}} + K_{P_3}(Q_{ref} - Q_{gen}) + x_3 - \frac{E_{qD}'}{X_m} - \frac{X_m}{X_r} I_{ds} \right\} \]

(2.60)

2.1.2 Zero-Axis Model (8th Order Model)

In this model, the dynamics of the rotor flux linkages is also assumed to be infinitely fast. Thus, the generator itself is modeled using algebraic equations (see next section where the steady-state model of the generator is presented). This assumption is supported by the fact that, in most cases, the time constant of the rotor flux linkage dynamics is still small compared with the time constants associated with the rest of the dynamics; i.e., \( \frac{1}{\omega_s} \ll \frac{2HD}{\omega_s} \) and \( \frac{1}{\omega_s} \ll 1 \) (see Equations (2.10)-(2.13) and (2.39)-(2.46)). To obtain the zero-axis model, define \( s = \frac{\omega - \omega_s}{\omega_s} \), which is called slip. Then, setting to zero the differential terms of Equations (2.10)-(2.13) and using Equations
Due to the alignment of the $d$-axis and the stator flux axis, the stator voltage has a zero angle and therefore $V_{qs} = V_D$ and $V_{ds} = 0$. With respect to the equation of motion, the electrical torque has to be expressed in terms of the model variables as

$$T_e = \psi_{dr} I_{qr} - \psi_{qr} I_{dr}$$
$$= (-X_m I_{ds} + X_r I_{dr}) I_{qr} - (-X_m I_{qs} + X_r I_{qr}) I_{dr}$$
$$= X_m (I_{qs} I_{dr} - I_{ds} I_{qr}) \quad (2.65)$$

Finally, the zero-axis model for the Type-C WTG is defined by the following set of DAEs:

**Differential Equations**

$$\frac{d\omega_r}{dt} = \frac{\omega_s}{2H_D} \left[ T_m - X_m I_{qs} I_{dr} + X_m I_{ds} I_{qr} \right] \quad (2.66)$$

$$\frac{dx_1}{dt} = K_{I1} [P_{ref} - P_{gen}] \quad (2.67)$$

$$\frac{dx_2}{dt} = K_{I2} [K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr}] \quad (2.68)$$

$$\frac{dx_3}{dt} = K_{I3} [Q_{ref} - Q_{gen}] \quad (2.69)$$

$$\frac{dx_4}{dt} = K_{I4} [K_{P3} (Q_{ref} - Q_{gen}) + x_3 - I_{dr}] \quad (2.70)$$

$$\frac{dx_5}{dt} = K_1 (\omega_r - \omega_{ref}) \quad (2.71)$$

$$\frac{dx_6}{dt} = x_5 - x_6 - \theta + K_P (\omega_r - \omega_{ref}) \quad (2.72)$$

$$\frac{d\theta}{dt} = x_6 \quad (2.73)$$
Algebraic Equations

0 = -V_{qr} + K_{P2} [K_{P1} (P_{ref} - P_{gen}) + x_1 - I_{qr}] + x_2  \quad (2.74)
0 = -V_{dr} + K_{P4} [K_{P3} (Q_{ref} - Q_{gen}) + x_3 - I_{dr}] + x_4  \quad (2.75)
0 = -P_{gen} + V_D I_{qs} - (V_{qr} I_{qr} + V_{dr} I_{dr})  \quad (2.76)
0 = -Q_{gen} + V_D I_{ds}  \quad (2.77)
0 = -V_D - R_s I_{qs} - X_s I_{ds} + X_m I_{dr}  \quad (2.78)
0 = -R_s I_{ds} + X_s I_{qs} - X_m I_{qr}  \quad (2.79)
0 = -V_{qr} + R_r I_{qr} - s X_m I_{ds} + s X_r I_{dr}  \quad (2.80)
0 = -V_{dr} + R_r I_{dr} + s X_m I_{qs} - s X_r I_{qr}  \quad (2.81)

Network Algebraic Equations

\[ V_{De} e^{j\theta_D} = (R + jX) (I_{qs} - jI_{ds} - I_{GC}) e^{j\theta_D} + V e^{j\theta}  \quad (2.82) \]

where \( I_{GC} \) is defined as

\[ I_{GC} = \frac{V_{qr} I_{qr} + V_{dr} I_{dr}}{V_D}  \quad (2.83) \]

Similarly to the two-axis model, the WTG and network algebraic equations can be alternatively represented by an equivalent circuit. The equivalent circuit is shown in Figure 2.9 and it is valid only when \( \omega_r \neq \omega_s \). More details about this equivalent circuit are presented in the next section. Note that Equations (2.74)-(2.81) fully express the rotor voltages in terms of the state and algebraic variables that belong to the equivalent circuit.

Figure 2.9: Equivalent circuit of the Type-C WTG connected to an infinite bus when the generator is represented by its zero-axis model.
2.2 Steady-State Model

This model is focused on the DFIG without considering controllers and the turbine model. Thus, mechanical torque, and \( q \) and \( d \)-axis rotor voltages are considered as inputs. Basically, to obtain this model, the dynamics of the stator and rotor flux linkages are assumed to be infinitely fast as in the zero-axis model. The equivalent circuit of Figure 2.9 can be used as a steady-state model, but a more practical representation can be derived. Using Equations (2.28) and (2.61)-(2.64), the following equations are obtained:

\[
\bar{V}_{as} = V_{qs} - jV_{ds} \tag{2.84}
\]

\[
\bar{V}_{ar} = \frac{V_{qr}}{s} - j\frac{V_{dr}}{s} \tag{2.88}
\]

\[
\frac{\bar{V}_{as}}{s} = -R_s I_{qs} + X_m I_{ds} + X_r I_{dr} - jR_s I_{ds} - jX_s I_{qs} + jX_m I_{qr} \tag{2.85}
\]

\[
\frac{\bar{V}_{ar}}{s} = \frac{R_r}{s} I_{qr} + X_r I_{ds} + X_m I_{dr} - jR_r I_{dr} - jX_r I_{qr} + jR_r I_{ds} + jX_m I_{qr} \tag{2.89}
\]

\[
\frac{\bar{V}_{as}}{s} = -(R_s + jX_s) I_{as} + jX_m I_{ar} \tag{2.86}
\]

\[
\frac{\bar{V}_{ar}}{s} = \left( \frac{R_r}{s} + jX_r \right) I_{ar} - jX_m I_{as} \tag{2.90}
\]

Equations (2.87)-(2.91) define the steady-state model of the doubly-fed induction generator (see Figure 2.10).

An alternative equivalent circuit can be derived by explicitly representing the mechanical power that comes from the turbine and the power injected to the rotor circuit by the rotor-side converter. Consider the voltage polarity and current directions defined in Figure 2.10. Then, the active power that

![Figure 2.10: Steady-state equivalent model of the doubly-fed induction machine.](image)
crosses the airgap is the power injected by the source \( \frac{V_{ar}}{s} \) minus the losses in the resistor \( R_r \). Thus,

\[
P_{ag} = \mathfrak{Re} \left( \frac{V_{ar}T_{ar}^*}{s} \right) - \frac{R_r}{s} |T_{ar}|^2 \tag{2.92}
\]

where \( \mathfrak{Re} \) is real part. On the other hand, physically, the power that crosses the airgap is the mechanical power from the shaft plus the power injected to the slip rings minus the rotor losses. Thus,

\[
P_{ag} = P_m + \mathfrak{Re} \left( \frac{V_{ar}T_{ar}^*}{s} \right) - R_r |T_{ar}|^2 \tag{2.93}
\]

Comparing Equations (2.92) and (2.93), the following expression for the mechanical power is obtained:

\[
P_m = \left\{ P_r - R_r |T_{ar}|^2 \right\} \frac{(1 - s)}{s} \tag{2.94}
\]

where \( P_r = \mathfrak{Re}(\frac{V_{ar}T_{ar}^*}{s}) \) [32, 33]. Splitting both the rotor resistance and voltage, an alternative equivalent circuit is obtained as shown in Figure 2.11. Note that by energy conservation, \( P_s = P_m + P_r - P_l \Rightarrow P_{gen} = P_s - P_r = P_m - P_l \). Using Equation (2.94), the electrical torque is defined by

\[
T_e = \frac{P_m \text{ [p.u.]} }{w_r \text{ [p.u.]} } = \frac{\left\{ P_r - R_r |T_{ar}|^2 \right\} }{s} \tag{2.95}
\]

The efficiency (\( \eta \)) of the doubly-fed induction machine depends on whether the machine is acting as a generator or as a motor. As a motor, \( |P_{gen}| > |P_m| \). As a generator, \( |P_{gen}| \leq |P_m| \). Neglecting mechanical and stator losses, the

![Figure 2.11: Alternative steady-state equivalent model of the doubly-fed induction machine.](image)
efficiency is

$$\eta_{gen} = \frac{P_{gen}}{P_m} = \frac{P_m - P_l}{P_m} = \frac{P_r(1-s) - R_r |I_{ar}|^2 \frac{1}{s}}{(P_r - R_r |I_{ar}|^2) \frac{(1-s)}{s}}$$ (2.97)

For the particular case when the rotor circuit is short-circuited, i.e., $V_{ar} = 0$ as an induction machine,

$$\eta_{gen} = \frac{P_{gen}}{P_m} = \frac{1}{(1 - s)}$$ (2.98)

When the doubly-fed induction machine is acting as a motor, the efficiency is $\eta_{motor} = \frac{P_m}{P_{gen}} = \eta_{gen}^{-1}$.

In order to understand the effect of the rotor voltages, the active and reactive power given by the generator are obtained for two cases. In the first one, assume that $V_{dr} = 0$ while $V_{qr}$ is varied from -0.04 to +0.04 p.u. In the second one, assume that $V_{qr} = 0$ while $V_{dr}$ is varied from -0.04 to +0.04 p.u. Consider $V_{as} = 1e^{j0}$, $X_{ls} = 0.03$, $X_{lr} = 0.08$, $X_m = 3$ and $R_s = R_r = 0.01$ (all quantities are in per unit). Figures 2.12 and 2.13 show the results. Notice that solid-, dotted- and dashed-line correspond to positive, zero and negative voltages, respectively. In the stable region of the power characteristic—the region where the derivative of the power with respect to the slip is negative—increasing values of $V_{qr}$ shift the active-power curve upwards and the reactive-power curve to the right (Figure 2.12); i.e., more active power is injected to the grid, but the machine reactive-power absorption is increased in generator mode. On the other hand, increasing values of $V_{dr}$ raise the negative active-power-curve slope and shift the reactive-power curve upwards (Figure 2.13); i.e., considerable reactive power is injected to the grid and the active power is slightly increased in generator mode. These results show the capability of the machine to give reactive-power support. Additionally, a strong coupling in both pairs ($P_{gen}, V_{qr}$) and ($Q_{gen}, V_{dr}$) and a weak coupling in ($P_{gen}, V_{dr}$) and ($Q_{gen}, V_{qr}$) are observed (Figure 2.14). The design of doubly-fed induction-machine controllers is based on these coupling characteristics.
Figure 2.12: Active power output as a function of $V_{qr}$, with $V_{dr} = 0$.

Figure 2.13: Reactive power output as a function of $V_{dr}$, with $V_{qr} = 0$.

Figure 2.14: Coupling between rotor voltages and power output.
2.3 On Power Balance and Active Power Controller

A closer look at the rotor speed controller is necessary. This controller can alternatively be called \textit{active power controller} due to the use of a one-to-one correspondence between rotor speed and active power (see power reference in Figure 2.4). Assume that the wind speed remains within its limits; thus, no pitch angle controller is necessary. The pitch angle is assumed to be zero. Then, given a wind speed and rotor speed, the mechanical power and torque at the turbine are calculated as

\[ P_m = BC_p(\lambda, \theta)v_{\text{wind}}^3 \text{ [p.u.]} \]  \hspace{1cm} (2.99)

\[ T_m = B\omega_s C_p(\lambda, \theta)\frac{v_{\text{wind}}^3}{\omega_r} \text{ [p.u.]} \]  \hspace{1cm} (2.100)

where \( C_p \) is defined as in Equation (2.3). In Figure 2.15, these variables are shown when the wind speed varies between 8 and 12 m/s and the rotor speed varies between 200 and 460 rad/s. Tracing a curve through the maximum power points, a power reference is obtained which is used in the speed controller (red line). Assume that this reference is expressed as \( P_{\text{ref}} = Cv_{\text{wind}}^3 \) [p.u.], where \( C \) is measured in [s^3].

Consider that the machine is directly connected to an infinite bus. This bus has a unity voltage and a zero angle; thus, \( V_{qs} = 1 \) and \( V_{ds} = 0 \). Consider \( X_{ls} = 0.03 \) p.u., \( X_{lr} = 0.08 \) p.u., \( X_m = 3 \) p.u., \( R_s = R_r = 0.01 \) p.u., \( B = 1.321 \times 10^{-3} \text{ m}^3/\text{s}^3 \), \( C = 1.0592 \times 10^{-8} \text{ s}^3 \) and \( \lambda = D\frac{\omega}{v_{\text{wind}}} \) where \( D = 1/6 \text{ m} \). Also, assume that the generator has a unity power factor which implies that \( I_{ds} = 0 \). By inspection of Equations (2.66)-(2.81), the equilibrium point is found by solving the following set of nonlinear equations:

\[ 0 = +B\omega_s C_p(\lambda, \theta)\frac{v_{\text{wind}}^3}{\omega_r} - X_m I_{qs} I_{dr} \]  \hspace{1cm} (2.101)

\[ 0 = -V_{qs} - R_s I_{qs} + X_m I_{dr} \]  \hspace{1cm} (2.102)

\[ 0 = -X_m I_{qr} + X_s I_{qs} \]  \hspace{1cm} (2.103)

\[ 0 = -V_{qr} + R_r I_{qr} + \frac{(\omega_s - \omega_r)}{\omega_s} X_r I_{dr} \]  \hspace{1cm} (2.104)

\[ 0 = -V_{dr} + R_r I_{dr} + \frac{(\omega_s - \omega_r)}{\omega_s} X_m I_{qs} - \frac{(\omega_s - \omega_r)}{\omega_s} X_r I_{qr} \]  \hspace{1cm} (2.105)

\[ 0 = -V_{qs} I_{qs} + V_{qr} I_{qr} + V_{dr} I_{dr} + C\omega_r^3 \]  \hspace{1cm} (2.106)
Note that the only system’s input is the wind speed. The equilibrium point should be such that the mechanical power is maximized; i.e., for a given wind speed, the rotor speed and the mechanical power are defined by the power reference curve (red line of Figure 2.15). The equilibrium points are calculated when the wind speed is varied from 8 to 12 m/s. Note that the power balance is satisfied at every equilibrium point. As mentioned in Section 2.2, $P_m + P_r = P_s + P_l$ where $P_l$ includes the losses at the stator and rotor windings. Also, note that the mechanical torque can be calculated using the power sent through the airgap as

$$T_m = \frac{P_m \text{ [p.u.]} \omega_r \text{ [p.u.]} = \frac{P_{ag} \text{ [p.u.]} \omega_s \text{ [p.u.]} = P_{ag} \text{ [p.u.]} = \frac{\{P_r - R_r |T_{ar}|^2\}}{s}}}{2.107}$$

where $\omega_s = 1 \text{ p.u.}$ Thus,

$$P_s = P_{ag} - R_s(I_{qs}^2 + I_{dr}^2)$$  \hspace{1cm} (2.108)
\[ P_{\text{gen}} = P_s - P_r = P_{ag} - R_s(I_{qs}^2 + I_{dr}^2) - P_r \]  
\[ = \frac{P_r}{s} - \frac{R_r}{s}(I_{qr}^2 + I_{dr}^2) - R_s(I_{qs}^2 + I_{dr}^2) - P_r \]  
\[ = P_r \frac{1 - s}{s} - R_r \frac{1 - s}{s}(I_{qr}^2 + I_{dr}^2) - P_l = P_m - P_l \]  

In general, the mechanical power and the electrical power should be of the same order of magnitude. Now, take a closer look at the equilibrium point when \( v_{\text{wind}} = 12 \text{ m/s} \). Powers, torques and rotor speed at the equilibrium point are

\[
\begin{align*}
P_m &= 1.0002 \text{ p.u.} \\
P_l &= 0.0148 \text{ p.u.} \\
P_{ag} &= 0.8322 \text{ p.u.} \\
P_{\text{gen}} &= 0.9854 \text{ p.u.} \\
P_s &= 0.8254 \text{ p.u.} \\
T_m &= T_e = 0.8322 \text{ p.u.} \\
P_r &= -0.1600 \text{ p.u.} \\
\omega_r &= 453.1110 \text{ rad/s}
\end{align*}
\]

Using the speed controller reference, it is verified that \( P_{\text{ref}} = C\omega_r^3 = P_{\text{gen}} = 0.9854 \text{ p.u.} \). However, when \( v_{\text{wind}} = 12 \text{ m/s} \), the intended maximum mechanical power is 1.003 p.u. at \( \omega_r = 455.4 \text{ rad/s} \). Although the mechanical power extracted from the wind is very close to the intended power, the rotor speed has a more notorious difference. This is due to power losses because the controller compares a mechanical power reference, \( P_{\text{ref}} \), with an electrical power, \( P_{\text{gen}} \). In Figure 2.16, the mechanical power, blue solid line, and mechanical torque characteristic, black solid line, are shown when \( v_{\text{wind}} = 12 \text{ m/s} \). The blue and black dashed lines correspond to the equilibrium point of the mechanical power and the mechanical torque, respectively, for several wind speeds. The intersection of the solid and dashed lines defines the equilibrium point. The red line corresponds to the power reference of the speed controller which is equal to \( P_{\text{gen}} \) at the equilibrium point.

### 2.4 Initial Condition Calculation

Given \( V_{as} \), \( v_{\text{wind}} \) and \( \theta = 0 \) (pitch angle), calculate all WTG’s variables at the equilibrium point.

**Step 1.** Compute optimal values for \( P_m \) and \( \omega_r \). Use look-up curves. For example, if \( v_{\text{wind}} = v_{\text{wind}}^\circ \text{[m/s]} \), set \( P_m = P_m^{\text{max}} \) and \( \omega_r = \omega_r^* \) as shown in Figure 2.17.
Step 2. Verify electrical rotor speed limits.

a. If $\omega_r < \omega_{\text{min}}$, shut down the WTG, go to step 8.

b. If $\omega_r > \omega_{\text{max}}$, set $P_m = P_{\text{max}}^m$ and $\omega_r = \omega_{\text{max}}$. If you want to calculate the corresponding $\theta$ for these maximum power and speed values, go to step 3. Else, go to step 4.

c. If $\omega_{\text{min}} \leq \omega_r \leq \omega_{\text{max}}$, go to step 4.

Step 3. Calculate $\theta$ by solving $P_m = \frac{1}{2} \rho A_w C_p (\frac{2k R \omega_{\text{max}}}{\rho v_{\text{wind}}}, \theta) v_{\text{wind}}^3$.

Step 4. Calculate slip, $s = \frac{\omega_s - \omega_r}{\omega_s}$.

Step 5. Compute $T_m = \frac{P_m}{\omega_r \frac{2}{p}}$.

Step 6. Set $T_e = T_m$.

Step 7. Find $V_{dr}$ and $V_{qr}$. Consider the steady-state model of Figure 2.10. The electrical torque is calculated as

$$T_e = \frac{P_r - R_r |J_{ar}|^2}{s}$$
Figure 2.17: $P_m$ as a function of $\omega_r$ and $v_{\text{wind}}$.

where

$$P_r = V_{qr} I_{qr} + V_{dr} I_{dr}$$

$$|I_{ar}|^2 = I_{qr}^2 + I_{dr}^2$$

$$\Rightarrow T_e = \frac{V_{qr} I_{qr} + V_{dr} I_{dr} - R_r (I_{qr}^2 + I_{dr}^2)}{s} \quad (2.112)$$

From the equivalent circuit two other equations can be derived. Obtain a Thevenin equivalent of the stator source and impedance of the stator and magnetizing reactance. By Kirchhoff's voltage law,

$$\nabla_{Th} = \frac{j X_m}{R_s + j X_s} \nabla_{as}$$

$$R_{Th} + j X_{Th} = \frac{j X_m (R_s + j X_{\ell s})}{R_s + j X_s}$$

$$\Rightarrow \nabla_{Th} + \left( R_{Th} + j X_{Th} + \frac{R_r}{s} + j X_{\ell r} \right) (I_{qr} - j I_{dr}) = \frac{V_{qr} - j V_{dr}}{s}$$

where $X_s = X_{\ell s} + X_m$. Taking the imaginary and real parts,

$$\text{Re}\{\nabla_{Th}\} + \left( R_{Th} + \frac{R_r}{s} \right) I_{qr} + (X_{Th} + X_{\ell r}) I_{dr} = \frac{V_{qr}}{s} \quad (2.113)$$

$$\text{Im}\{\nabla_{Th}\} - \left( R_{Th} + \frac{R_r}{s} \right) I_{dr} + (X_{Th} + X_{\ell r}) I_{qr} = -\frac{V_{dr}}{s} \quad (2.114)$$

So far, there are three equations and four unknowns: $V_{qr}, V_{dr}$,
and $I_{dr}$. $V_{dr}$ is related to the reactive power generated by the DFIG. Thus, we can either specify $V_{dr} = 0$ or calculate $V_{dr}$ such that a certain reactive power is generated. In the first case, the DFIG will likely absorb reactive power from the infinite bus (terminal voltage). No extra equation is required. The three remaining variables can be computed.

In the second case, additional equations are required. By Kirchhoff’s voltage law in the stator circuit,

$$V_{qs} - jV_{ds} = -(R_s + jX_s)(I_{qs} - jI_{ds}) + jX_m(I_{qr} - jI_{dr})$$

\[ \Rightarrow V_{qs} + R_sI_{qs} + X_sI_{ds} - X_mI_{dr} = 0 \]  \hspace{1cm} (2.115)

$$-V_{ds} + X_sI_{qs} - R_sI_{ds} - X_mI_{qr} = 0$$

The final equation is derived from the reactive power equation at the stator as

$$Q_{ref} = Q_s = \text{Im}\{V_{as}T_{as}^*\} = \text{Im}\{(V_{qs} - jV_{ds})(I_{qs} + jI_{ds})\}$$

\[ \Rightarrow Q_{ref} - V_{qs}I_{ds} + V_{ds}I_{qs} = 0 \]  \hspace{1cm} (2.117)

Finally, there are six unknowns ($V_{qr}, V_{dr}, I_{qr}, I_{dr}, I_{qs}, I_{ds}$) and six equations.

Step 8. End.

2.5 Capability Curve

The reactive-power capability of a doubly-fed induction machine presents similarities to the conventional synchronous generator capability. It depends on the active power generated, the slip, and the limitations due to stator and rotor maximum currents as well as the maximum rotor voltage [34, 35]. In order to understand the power capability curve, the following circuital
relationships are obtained from the equivalent circuit (Figure 2.10):

\[
\begin{bmatrix}
\mathbf{V}_{as} \\
\mathbf{V}_{ac_s}
\end{bmatrix} = 
\begin{bmatrix}
-(R_s + jX_s) & jX_m \\
-jX_m & R_s + jX_r
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_{as} \\
\mathbf{I}_{ar}
\end{bmatrix}
\begin{bmatrix}
\mathbf{Z}
\end{bmatrix}
\]  
(2.118)

\[
\frac{\mathbf{I}_{as}}{\mathbf{I}_{ar}} = \mathbf{Y} \begin{bmatrix}
\mathbf{V}_{as} \\
\mathbf{V}_{ac_s}
\end{bmatrix}, \quad \mathbf{Y} = \mathbf{Z}^{-1}
\]  
(2.119)

\[
\begin{bmatrix}
\mathbf{I}_{as} \\
\mathbf{V}_{as}
\end{bmatrix} = 
\begin{bmatrix}
\frac{-1}{R_s + jX_s} & jX_m \\
\frac{jX_m}{R_s + jX_s} & \frac{R_r + jX_r}{R_s + jX_s}
\end{bmatrix}
\begin{bmatrix}
\mathbf{V}_{as} \\
\mathbf{I}_{ar}
\end{bmatrix}
\begin{bmatrix}
\mathbf{G}
\end{bmatrix}
\]  
(2.120)

\[
\begin{bmatrix}
\mathbf{V}_{as} \\
\mathbf{I}_{ar}
\end{bmatrix} = 
\begin{bmatrix}
\frac{R_r + jX_r}{jX_m} & \frac{(R_r + jX_r)(R_s + jX_s) + X_m^2}{jX_m} \\
\frac{jX_m}{R_r + jX_r} & \frac{R_r + jX_r}{jX_m}
\end{bmatrix}
\begin{bmatrix}
\mathbf{V}_{as} \\
\mathbf{I}_{as}
\end{bmatrix}
\begin{bmatrix}
\mathbf{B}
\end{bmatrix}
\]  
(2.121)

2.5.1 Maximum Rotor Current

Considering that the converters and the DC-link are lossless and the grid-side converter is typically operated at unity power factor, the total power injected by the doubly-fed induction generator is \( S_{gen} = S_s - \Re \{ S_r \} \), where \( S_s \) is the complex power going out of the stator and \( S_r \) is the complex power going into the rotor. Using Equation (2.120), these powers are defined by

\[
\overline{S}_s = \overline{V}_{as}\overline{I}_{as} = \overline{V}_{as}(G_{11}\overline{V}_{as} + G_{12}\overline{I}_{ar})^* = \frac{-1}{R_s - jX_s} |\overline{V}_{as}|^2 + \frac{-jX_m}{R_s - jX_s} \overline{V}_{as}\overline{I}_{ar}^* \tag{2.122}
\]

\[
\overline{S}_r = \overline{V}_{ar}\overline{I}_{ar} = s(G_{21}\overline{V}_{as} + G_{22}\overline{I}_{ar})\overline{I}_{ar} = \frac{s jX_m}{R_s + jX_s} \overline{V}_{as}\overline{I}_{ar}^* + s \left( \frac{R_r}{s} + jX_r + \frac{X_m^2}{R_s + jX_s} \right) |\overline{I}_{ar}|^* \tag{2.123}
\]

Typically, \( R_s \ll X_m < X_s \) and \( R_r \ll X_m < X_r \). Thus, the real part of the second term on the right-hand side of Equation (2.123) may be neglected. It turns out that \( P_r = \Re (\overline{S}_r) \approx s \Re (\overline{S}_s) = sP_s \). In Figure 2.18, a scheme of the active-power balance under super- and sub-synchronous speed is presented. \( P_m \) corresponds to the mechanical power sent into the machine shaft by the wind turbine. Note that the imaginary part of Equation (2.123),
which corresponds to the reactive power injected to the rotor, is supplied by the rotor-side converter instead of the grid. In summary, $\mathcal{S}_{gen} = \mathcal{S}_s - sP_s$.

Assuming $V_{as} = V_s e^{j\theta}$, $I_{ar} = I_r e^{-j\theta}$ and $R_s \ll X_s$, a simplified expression for the complex power injected by the generator is obtained as follows:

$$
\mathcal{S}_{gen} \approx \frac{1}{jX_s} |V_{as}|^2 + \frac{X_m X_{as} I_{ar}^*}{X_s} - s \Re \mathcal{S}_s \\
\approx \frac{|V_{as}|^2}{jX_s} + \frac{X_m}{X_s} V_{as} I_{ar}^* - s \Re \left\{ \frac{X_m}{X_s} V_{as} I_{ar}^* \right\} \\
= -j \frac{V_{s}^2}{X_s} + (1 - s) \frac{X_m}{X_s} V_s I_r \cos \theta + j \frac{X_m}{X_s} V_s I_r \sin \theta
$$

Equation (2.124) represents an ellipsoid with center at $-j \frac{V_{s}^2}{X_s}$ in the $P - Q$ plane. If $I_r = I_{r, max}$, then this ellipsoid defines the boundary of a safe operation.

2.5.2 Maximum Rotor Voltage and Stator Current

Similar to the maximum rotor-current limitation, boundaries of a safe operation can be found by considering a maximum rotor voltage and maximum stator current. For maximum rotor voltage, it is required to find an expression for the relationship between power and rotor voltage. Use Equation (2.119) to obtain

$$
\mathcal{S}_s = V_{as} I_{as}^* = V_{as} \left( Y_{11} V_{as} + Y_{12} \frac{V_{ar}}{s} \right)^*
$$

$$
\mathcal{S}_r = V_{ar} I_{ar}^* = V_{ar} \left( Y_{21} V_{as} + Y_{22} \frac{V_{ar}}{s} \right)^*
$$

Figure 2.18: Active power flows in the doubly-fed induction machine.
For the stator current, an expression for the relationship between power and stator current is required. Use Equation (2.121) to obtain

\[ \bar{S}_s = \bar{V}_{as} \bar{I}_{as} \]  
\[ \text{(2.127)} \]

\[ \bar{S}_r = \bar{V}_{ar} \bar{I}_{ar} = s (\bar{B}_{11} \bar{V}_{as} + \bar{B}_{12} \bar{I}_{as}) (\bar{B}_{21} \bar{V}_{as} + \bar{B}_{22} \bar{I}_{as})^* \]  
\[ \text{(2.128)} \]

For each case compute \( S_{gen} = \bar{S}_s - \Re(\bar{S}_r) \). In Figure 2.19, the capability curve considering super-synchronous is presented. The same parameters used in Section 2.2 are considered. Among the three limits described above, at every point the lowest one is used to create the capability curve (thick solid line).

![Figure 2.19: Power capability of a doubly-fed induction generator at super-synchronous speed with \( V_s = 1 \) p.u., \( s = -0.2 \), \( I_s^{max} = 1 \) p.u., \( I_r^{max} = 1 \) p.u., \( V_r^{max} = 0.24 \) p.u. and \( P^{max} = 0.6 \) p.u.](image)

2.6 Reactive Power Control versus Voltage Control

The main advantage of the Type-C WTG over the Type-A and Type-B WTG is that it can operate with variable angular speed, allowing it to extract maximum power from the wind. In addition, having a capability curve similar to synchronous generators, the Type-C WTG can inject a limited but controllable amount of reactive power to the grid (Type-A and Type-B WTG only absorb reactive power). It seems that a Type-C WTG can be considered as
a synchronous generator from a power flow point of view (PV bus). However, there are characteristics that make Type-C WTG not exactly equal to synchronous generators. First of all, in wind farms there is a huge accumulation of turbines producing varying values of power while traditional power plants that use synchronous generators typically consider a few generators, say 4 to 10, which are relatively close together. Secondly, the capability curve in Type-C WTG is more restrictive than in synchronous generators. Therefore, the reactive power injection from Type-C WTG is moderate in comparison with the reactive power that a synchronous generator can provide. These characteristics make the modeling of Type-C WTGs different than synchronous generators. In order to support this assertion, consider a wind farm of 20 WTGs (Appendix I). The active power is defined by using the wind speeds considered in the previous section. Assume $S_b = 100$ MVA. Two cases are presented [36]. In the first one, every WTG in the wind farm is designed to keep a unity terminal voltage, and in the second case, every WTG is designed to keep a unity power factor (controlling reactive power with a set-point equal to zero). The power flow solutions for these two cases are presented in Figures 2.20 and 2.21. The flow directions of the active and reactive power are indicated by green and blue arrows, respectively. Observe that while in the first case the wind farm absorbs an important amount of reactive power from the grid (22.89 Mvar), in the second case, the react-

![Figure 2.20: Wind farm with WTGs keeping a unity terminal voltage.](image)

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Active power absorption is considerably reduced (2.98 Mvar). The wind farm operation in the first case is odd. All WTGs, except WTG 6 and 11, absorb reactive power to keep the desired voltage (1 p.u.). Note that in the second case, although no WTG is producing reactive power, there are still many buses with overvoltage. There is no need for reactive power injection. The simple facts that there are many sources (WTGs) injecting power in the wind farm network and that the wind farm terminal voltage is imposed by the grid raise the voltage magnitude at the internal buses. The overloading of WTGs in the case of voltage control and the over-voltage in the case of reactive power control can be explained by using phasor diagrams. Consider a single WTG connected through a line to an infinite bus as shown in Figure 2.22.

In order to consider a weaker connection of the wind farm with the grid, modify the parameters of the line between bus 1 and the PCC to $R = 0.08$ p.u., $X = 0.65$ p.u. and $B = 0.08$ p.u. In this case, the voltage at bus 1, and consequently the internal buses of the wind farm network, are notably reduced. The transmission of the power produced by the wind farm can be improved by increasing the voltage at bus 1. Reactive power is needed, and an interesting question is: Where should this reactive power be provided? Should the individual WTGs provide the reactive power that the system needs? To find the answer, four scenarios of reactive power injections are...
Figure 2.22: Phasor voltage diagram of a WTG connected to an infinite bus: (a) WTG keeps its terminal voltage to unity, (b) WTG keeps a zero reactive power injection.

considered. Also, a reactive power compensator is used at bus 2. In the first two cases the compensator is inactive. Voltage control with $V_{\text{ref}} = 1 \text{ p.u.}$ and reactive power control with $Q_{\text{ref}} = 0 \text{ [Mvar]}$ at every WTG are considered, respectively. In the last two cases, the compensator is taken into account and once again voltage control with $V_{\text{ref}} = 1 \text{ p.u.}$ and reactive power control with $Q_{\text{ref}} = 0 \text{ Mvar}$ at every WTG are considered, respectively. In Table 2.1, the results are presented. The following comments about the voltage and reactive power control can be inferred from the results:

a. Observe that even when the wind farm is lacking reactive power, voltage control does not provide adequate wind farm operation. WTGs are overloaded due to the absorption of reactive power. In addition, the WTGs 6 and 11 have to provide an important amount of reactive power to keep the voltage profile (1 p.u.). Note that the reactive power given by WTG 6 when the compensator is inactive is prohibited. When the compensator is activated no improvement is observed. The absorption of reactive power by WTGs is kept at the same value as when the compensator is inactive. Only WTGs 6 and 11 reduce their reactive power injection due to the presence of the compensator. Observe that the power provided by the compensator is relatively high. In addition, note that the system losses without and with the compensator are 15.64 MW and 16.14 MW, respectively.
b. Using reactive power control with $Q_{\text{ref}} = 0$ at every WTG, the voltage profile in the wind farm network is not critical, but there is undervoltage. When the compensator is activated, there is overvoltage. However, the voltage profile does not seem to be critical. The losses without and with the compensator are 10.8 MW and 8.92 MW, respectively.

Table 2.1: Four scenarios for voltage and reactive power control in Wind Farm A (20 WTGs)

<table>
<thead>
<tr>
<th>Element</th>
<th>No compensator</th>
<th>With compensator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q$ (V Control)</td>
<td>$V$ (Q Control)</td>
</tr>
<tr>
<td>WTG1</td>
<td>-3.30</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG2</td>
<td>-15.39</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG3</td>
<td>-7.59</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG4</td>
<td>-8.24</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG5</td>
<td>-11.17</td>
<td>0.93</td>
</tr>
<tr>
<td>WTG6</td>
<td>105.38</td>
<td>0.90</td>
</tr>
<tr>
<td>WTG7</td>
<td>-3.88</td>
<td>0.91</td>
</tr>
<tr>
<td>WTG8</td>
<td>-5.17</td>
<td>0.92</td>
</tr>
<tr>
<td>WTG9</td>
<td>-9.15</td>
<td>0.93</td>
</tr>
<tr>
<td>WTG10</td>
<td>-11.23</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG11</td>
<td>10.51</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG12</td>
<td>-2.98</td>
<td>0.94</td>
</tr>
<tr>
<td>WTG13</td>
<td>31.44</td>
<td>0.95</td>
</tr>
<tr>
<td>WTG14</td>
<td>-8.15</td>
<td>0.95</td>
</tr>
<tr>
<td>WTG15</td>
<td>-9.11</td>
<td>0.95</td>
</tr>
<tr>
<td>WTG16</td>
<td>-3.29</td>
<td>0.96</td>
</tr>
<tr>
<td>WTG17</td>
<td>-10.54</td>
<td>0.96</td>
</tr>
<tr>
<td>WTG18</td>
<td>-8.53</td>
<td>0.97</td>
</tr>
<tr>
<td>WTG19</td>
<td>-9.71</td>
<td>0.97</td>
</tr>
<tr>
<td>WTG20</td>
<td>-11.27</td>
<td>0.98</td>
</tr>
<tr>
<td>Compensator</td>
<td>–</td>
<td>0.99</td>
</tr>
</tbody>
</table>

In conclusion, controlling voltage in the WTGs of a wind farm instead of controlling reactive power is not recommended. When voltage control is used, WTGs absorb a high amount of reactive power, the reactive power
absorption from the grid is relatively high and the wind farm’s losses are higher than when reactive power control is employed. It seems that the distributed generation in the wind farm’s network raises the bus voltage in the network without requiring any further control. Thus, from a power flow point of view, the terminals of the WTGs should be considered as PQ buses instead of PV buses. In the case of having a compensator controlling voltage at bus 2, then the modeling of the wind farm can be performed in two blocks: one block for modeling the active power coming from the wind and an independent block modeling the voltage imposed by the compensator. Still, the buses of the WTGs should be considered as PQ buses and only the compensator should be represented as a PV bus.

2.7 Modal Analysis

Consider the 4-bus system of Figure 2.23. Assume an exponential model for the load as $P_L = P_0V_L^{p_l}$ and $Q_L = Q_0V_L^{q_l}$. The SG is represented by a two-axis model [29] and the transient reactance at both $q$- and $d$-axis are equal, i.e., $X_d' = X_q'$. An IEEE Type-1 exciter and a linear speed governor without droop [37] are considered. The WTG is represented by its two-axis model. Assume that the wind speed is within its limits (cut-in and maximum speed). Consequently, no pitch angle controller is required and the rotor speed controller is operating over the optimal tracking curve. This curve is defined as $P_{ref} = C\omega_r^3$ p.u. The system data is shown in Appendix B. The complete set of differential algebraic equations for this system is presented in Appendix C.

Figure 2.23: Four-bus test system. One-line diagram and circuitual representation.
Assume a wind speed of 12 m/s and a loading of $P_0 = 0.9$ p.u. and $Q_0 = 0.18$ p.u. The corresponding system equilibrium point is stable and the eigenvalues with their oscillatory frequency, $f_{osc}$, and damping ratio, $\sigma$, are listed in Table 2.2 (see Appendix D for more details). Using participation factors [29], the state variables associated with the modes are presented. Note that given a generic complex eigenvalue $\mu = \mu_x + j\mu_y$, the oscillatory frequency and damping ratio are defined by

$$f_{osc} = \frac{|\mu_y|}{2\pi} \quad \sigma = \frac{-\mu_x}{\sqrt{\mu_x^2 + \mu_y^2}} \quad (2.129)$$

Moreover, assume that $v$ and $w$ are the right and left eigenvectors of $\mu$. Then, the participation factor of the $i^{th}$ state variable in mode $\mu$ is [29]

$$p_{i,\mu} = \frac{|w_i||v_i|}{\sum_{j=1}^{n} |w_j||v_j|} \quad (2.130)$$

where $n$ is the total number of state variables and $w_i$ and $v_i$ correspond to the $i^{th}$ component of the left and right eigenvector, respectively. The presented state variables are those with a participation factor greater than 0.05 (Table 2.2). Note that the eigenvalues related to the WTG variables lie on the negative real axis; thus, after a small perturbation, these variables will

Table 2.2: Eigenvalues for a wind speed of 12 m/s and a loading of $P_0 = 0.9$ p.u., $Q_0 = 0.18$ p.u.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>$f_{osc}$</th>
<th>$\sigma$</th>
<th>State variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = -1818.3$</td>
<td>0</td>
<td>1</td>
<td>$E'_q, E'_d, E'_dD$</td>
</tr>
<tr>
<td>$\mu_2 = -566.6$</td>
<td>0</td>
<td>1</td>
<td>$E'_q, E'_d, E'_dD$</td>
</tr>
<tr>
<td>$\mu_3 = -13.4$</td>
<td>0</td>
<td>1</td>
<td>$E'_q, E'_d, V_R$</td>
</tr>
<tr>
<td>$\mu_{4,5} = -5.41 \pm j3.33$</td>
<td>0.530</td>
<td>0.851</td>
<td>$E'_q, E'_d, E'_d, R_f, V_R$</td>
</tr>
<tr>
<td>$\mu_{6,7} = -4.23 \pm j2.90$</td>
<td>0.461</td>
<td>0.825</td>
<td>$E'_q, E'_d, E'_d, E'_fd, R_f$</td>
</tr>
<tr>
<td>$\mu_8 = -5.10$</td>
<td>0</td>
<td>1</td>
<td>$E'_d, x_2, x_4$</td>
</tr>
<tr>
<td>$\mu_{9,10} = -0.02 \pm j1.94$</td>
<td>0.308</td>
<td>0.010</td>
<td>$\omega, P_m$</td>
</tr>
<tr>
<td>$\mu_{11} = -2.38$</td>
<td>0</td>
<td>1</td>
<td>$x_1, x_3$</td>
</tr>
<tr>
<td>$\mu_{12} = -2.05$</td>
<td>0</td>
<td>1</td>
<td>$x_1, x_2, x_3, x_4, E'_q, R_f$</td>
</tr>
<tr>
<td>$\mu_{13} = -1.24$</td>
<td>0</td>
<td>1</td>
<td>$x_3, E'_d, R_f$</td>
</tr>
<tr>
<td>$\mu_{14} = -0.08$</td>
<td>0</td>
<td>1</td>
<td>$\omega_r$</td>
</tr>
</tbody>
</table>
exhibit no oscillatory behavior. In addition, observe that there is no direct coupling between the DFIG’s speed and SG’s speed—\(\mu_9,10\) and \(\mu_{14}\). Note that the eigenvalues associated with \(E'_{qD}\) and \(E'_{dD}\) are far to the left, which shows that the dynamics of these voltages are much faster than the dynamics of other states. Thus, the dynamics of \(E'_{qD}\) and \(E'_{dD}\) can be neglected obtaining the already defined zero-axis model.

In order to study the system behavior for different operating points, a bifurcation analysis is performed [38] (see Appendix E for more details). The load is varied from \(P_0 = 0.32\) p.u. to \(P_0 = 1\) p.u. keeping a constant power factor; i.e., the ratio \(\frac{Q}{P_0}\) is kept constant. For wind speeds of 10 [\text{m/s}] and 12 m/s, the trajectories of the dominant eigenvalues are shown in Figure 2.24. The pathway of eigenvalues is subtly different for the two presented cases. This difference is related to parameter sensitivities [39] due to the increase of the power generated by the WTG. Note that the eigenvalues associated with \(\omega_r, \omega\) and \(P_m\) do not move when load and wind speed are varied. The eigenvalues that cross the imaginary axis are associated with states \(E'_q, E'_d, E_{fd}\) and \(V_R\). The HB points for these two cases occur at a loading of \(P_0 = 0.942\) p.u. and \(P_0 = 0.972\) p.u., respectively. This does not differ considerably from the modal behavior of a system with a conventional synchronous generator [29,33,40]. For all wind speeds and loadings simulated, eigenvalues associated with the WTG’s state-variables are generally stable. After the system hits the HB point, the unstable pair of complex eigenvalues

![Diagram](image)

Figure 2.24: Dominant eigenvalues trajectory for wind speeds of 10 m/s and 12 m/s.
coalesces on the real line and then splits into two real eigenvalues moving in opposite directions. The one moving to the right is associated with the DFIG’s internal voltages, i.e., $E'_{qD}$ and $E'_{dD}$.

It seems that the WTG dynamics have no serious impact on system behavior. Probably the most significant interaction between the machines is the interchange of active power. Consequently, a negative load might be sufficient to represent WTGs in power system analysis. To verify this assertion, a comparison of the WTG with other models is executed. The wind speed is assumed to be constant and equal to 12 m/s. Note that in steady state, the total active power injected by the WTG is approximately 0.298 p.u. A unity power factor is considered.

### 2.7.1 Alternative Wind Power Representation

Synchronous rather than induction machines are generally used in power systems. Given this, it would be interesting to see the system response if the wind power were represented by a synchronous machine. Thus, the WTG is replaced by a hypothetical synchronous generator (HSG). A two-axis model, a linear speed governor with droop [29], and a reactive-power controller (a PI-controller with an IEEE Type-1 exciter as shown in Figure 2.25) are considered. The data of the machine and the controllers is the same as for the SG, except that $H = 4$ s and $R_D = 5\%$ (frequency droop). For the PI-controller, $K_{I5} = 0.15$ p.u. and $K_{P5} = 0.5$ p.u. The set points for the governor and the reactive-power controller are 0.298 p.u. and 0 p.u., respectively. Additionally, the WTG is also replaced by a negative load (NL) with $P_{NL} = -0.298$ p.u. and $Q_{NL} = 0$ p.u.

The trajectory of dominant eigenvalues for each case is shown in Figure

![Figure 2.25: Reactive-power controller of the hypothetical synchronous generator.](image)
Additionally, the HB points and state variables associated with the unstable modes are shown in Table 2.3. The load is varied from $P_0 = 0.32$ p.u. to $P_0 = 1.048$ p.u. System behavior is similar when the wind power is represented using either the WTG or the NL model. Moreover, loadings at the HB point for these two cases are identical. Surprisingly, the HSG behavior is radically different. More dominant eigenvalues are observed and the modes which become unstable have a lower frequency. The HB point is not even close to the one calculated in the base case. Some HSG variables are associated with the unstable modes. This does not happen when the WTG or the NL model is used.

The differences between the HSG and WTG may be related to frequency response. In the HSG case, there is a steady-state frequency droop causing a greater interaction with the SG. The WTG has no such frequency response; the oscillations in its rotor are mainly associated with wind speed variations and set-point adjustments. Note that there are wind farm configurations with frequency response [25]. Basically, the wind turbines are operated to extract not maximum power from the wind, but rather a fraction of it, say 80%. This allows a power cushion, and the wind farm can supply more or less power when there is a frequency deviation. The behavior of wind generators with frequency response remains to be analyzed.

The eigenvalues pathways for wind speeds of 12 and 10 m/s are presented.

![Figure 2.26: System eigenvalue trajectories considering a WTG, a synchronous generator and a negative load. Active power output of 0.298 p.u.](image)

Figure 2.26: System eigenvalue trajectories considering a WTG, a synchronous generator and a negative load. Active power output of 0.298 p.u.
Table 2.3: Load at the system HB point

<table>
<thead>
<tr>
<th>Model</th>
<th>Loading at HB point</th>
<th>Variables associated with unstable modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTG</td>
<td>0.972</td>
<td>SG: $E'_q$, $E'<em>d$, $E</em>{fd}$, $V_R$</td>
</tr>
<tr>
<td>HSG</td>
<td>1.034</td>
<td>SG: $E'<em>q$, $E</em>{fd}$, $R_f$; HSG: $E'_q$, $E'_d$</td>
</tr>
<tr>
<td>NL</td>
<td>0.972</td>
<td>SG: $E'_q$, $E'<em>d$, $E</em>{fd}$, $V_R$</td>
</tr>
</tbody>
</table>

in Figures 2.27 and 2.28 when the WTG and NL models are used. Also, the movement direction of the modes when the load is increased and the corresponding modes when the loading is $P_0 = 0.9$ p.u. and $P_0 = 0.95$ p.u. are shown. Note that although the modes obtained using the WTG and the NL model are unlike for loading less than 0.9 p.u., these are very similar for higher loading, especially those modes that cross the imaginary axis.

2.7.2 Sensitivity Analysis

To verify whether or not an NL model suffices to represent WTG in power system analysis, sensitivity analysis related to several parameters is performed.

Reactive-power set-point

$Q_i$ is the injected reactive power at bus $D$; thus, $Q_{ref} = Q_i$ for the WTG and $Q_{NL} = -Q_i$ for the NL. In Table 2.4, the loading and the complex pair of eigenvalues at the HB point are presented when $Q_i$ is varied from $-0.2$ to $+0.2$ pu. The values separated by the slash correspond to the WTG and the NL, respectively. Observe that using the NL, $f_{osc}$ is an upper bound for the base case. Moreover, using the NL, estimation of the HB point suffices, i.e., the values are close to the base case, just a little higher or lower depending on whether the reactive power is injected or absorbed.

WTG inertia

The inertia has minimal influence on the eigenvalues. The inertia is varied from 0.5 to 10 s. Dominant eigenvalue trajectories show almost no change. A slight decrease in loading is observed when the inertia is increased. Note that inertia is more influential during transient operation. The rotational motion
Figure 2.27: Dominant eigenvalues pathway using the WTG model for $v_{\text{wind}} = 12 \text{ m/s}$ and $v_{\text{wind}} = 10 \text{ m/s}$.

Figure 2.28: Dominant eigenvalues pathway using the NL model for $P_{NL} = -0.2980 \text{ p.u.}$ and $P_{NL} = -0.1723 \text{ p.u.}$.

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Table 2.4: HB point sensitivity with respect to $Q_i$

<table>
<thead>
<tr>
<th>$Q_i$</th>
<th>Loading at HB point</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.039 / 1.045</td>
<td>$\pm j7.10 / \pm j7.17$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.007 / 1.010</td>
<td>$\pm j7.88 / \pm j8.01$</td>
</tr>
<tr>
<td>0</td>
<td>0.972 / 0.972</td>
<td>$\pm j8.60 / \pm j8.69$</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.935 / 0.932</td>
<td>$\pm j9.06 / \pm j9.37$</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.897 / 0.889</td>
<td>$\pm j9.42 / \pm j9.83$</td>
</tr>
</tbody>
</table>

of a WTG acts as a filter of wind speed fluctuations. Consequently, if the generator has a lower inertia, its power output would present more fluctuations, causing more oscillations in the system’s synchronous generators.

Static load’s parameters

Load modeling is complex because it is dependent on patterns of people’s behavior which are difficult to define. So far, it has been considered as an exponential load model with $pv = qv = 0$ (static constant power load). Due to load-model uncertainty, three new loads are used: an induction motor (IM) at half load, an IM at full load, and a room air conditioner (RAC) [41, 42]. In Table 2.5, the loading at the HB point and its associated pair of complex eigenvalues are presented. The results show a great agreement between the WTG and NL models.

Network’s parameters

Generally, wind power generation is located either on the sub-transmission or distribution side. In these cases, the network has an important resistance which may impact the analysis. The parameters of line 1 (Figure 2.23), which represent the sub-transmission side, will be doubled in order to account for a weaker connection to the grid. The resistance of line 2, distribution side, will be varied between 0.05 and 0.15 p.u. Once again, the results (Table 2.6) show a good agreement between the WTG and NL models.
Table 2.5: HB point and load parameters dependence

<table>
<thead>
<tr>
<th>Load</th>
<th>pv</th>
<th>qv</th>
<th>Loading at HB point</th>
<th>µ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0</td>
<td>0</td>
<td>0.972 / 0.972</td>
<td>±j8.60 / ±j8.69</td>
</tr>
<tr>
<td>IM half load</td>
<td>0.2</td>
<td>1.6</td>
<td>1.185 / 1.187</td>
<td>±j7.86 / ±j7.78</td>
</tr>
<tr>
<td>IM full load</td>
<td>0.1</td>
<td>0.6</td>
<td>1.068 / 1.069</td>
<td>±j8.23 / ±j8.26</td>
</tr>
<tr>
<td>RAC</td>
<td>0.5</td>
<td>2.5</td>
<td>1.499 / 1.503</td>
<td>±j7.52 / ±j7.16</td>
</tr>
</tbody>
</table>

Table 2.6: HB point and network parameters dependence

<table>
<thead>
<tr>
<th>R₁ + jX₁</th>
<th>R₂ + jX₂</th>
<th>Loading at HB point</th>
<th>µ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03+j0.1</td>
<td>0.05+j0.1</td>
<td>1.101 / 1.101</td>
<td>±j8.31 / ±j8.28</td>
</tr>
<tr>
<td>0.03+j0.1</td>
<td>0.1+j0.1</td>
<td>0.972 / 0.972</td>
<td>±j8.60 / ±j8.69</td>
</tr>
<tr>
<td>0.03+j0.1</td>
<td>0.15+j0.1</td>
<td>0.861 / 0.861</td>
<td>±j9.04 / ±j9.42</td>
</tr>
<tr>
<td>0.06+j0.2</td>
<td>0.05+j0.1</td>
<td>0.895 / 0.897</td>
<td>±j9.30 / ±j9.08</td>
</tr>
<tr>
<td>0.06+j0.2</td>
<td>0.1+j0.1</td>
<td>0.808 / 0.809</td>
<td>±j9.61 / ±j9.63</td>
</tr>
<tr>
<td>0.06+j0.2</td>
<td>0.15+j0.1</td>
<td>0.730 / 0.731</td>
<td>±j9.80 / ±j10.28</td>
</tr>
</tbody>
</table>

Active and reactive power controllers’ parameters

The tuning of PI controllers can be considered an art which requires experience and a detailed knowledge of the system under control. There are many tuning rules but usually the parameters must be adjusted through trial and error simulations [43]. Thus, proportional and integral parameters of the active and reactive power controllers are varied to observe the incidence of each of them on the system’s eigenvalues. To consider this variation, a multiplicative factor is considered as $K_{P,i} = \text{factor} \times K_{P,i,\text{base}}$ and $K_{I,i} = \text{factor} \times K_{I,i,\text{base}}$, where $i = \{1, 2, 3, 4\}$. $K_{P,i,\text{base}}$ and $K_{I,i,\text{base}}$ correspond to the proportional and integral gains for the base case. It is observed that eigenvalues are practically insensitive to $K_{I1}$, $K_{I3}$, $K_{P1}$ and $K_{P3}$ variations which are related to the slow loop of the active and reactive power controllers. With respect to the parameters of the fast loop of the controllers, two cases are considered. In the first one, $K_{I2} = \text{factor} \times K_{I2,\text{base}}$ and $K_{P2} = \text{factor} \times K_{P2,\text{base}}$ while $K_{I4}$ and $K_{P4}$ are kept at their base values (Figure 2.29). In the second case, $K_{I4} = \text{factor} \times K_{I4,\text{base}}$ and $K_{P4} = \text{factor} \times K_{P4,\text{base}}$ while $K_{I2}$ and $K_{P2}$ are kept at their base values (Figure 2.30). In the first case, lower parameters increase the oscillation
Figure 2.29: Eigenvalues sensitivity with respect to $K_{P2}$ and $K_{I2}$.

Figure 2.30: Eigenvalues sensitivity with respect to $K_{P4}$ and $K_{I4}$.
frequency of the modes. Higher parameters do not considerably decrease the frequency of the modes with respect to the base case. In the second case, an opposite behavior is observed. Lower parameters can decrease the oscillation frequency of the modes, making the system more stable. Parameters above those of the base case do not considerably modify the system’s eigenvalues. Remember that these parameters are related to the fast loop of the reactive power control and the set-point is set to zero. Thus, just an adjustment of these parameters can be beneficial for the system’s stability without requiring more power capability of the WTG. The loading at the HB point is 0.962 p.u. when factor=0.1. For other factor values, the loading at the HB point remains approximately the same (0.972 p.u.).

Note that when the eigenvalues’ pathway differs from that of the base case, due to a parameter adjustment, some WTG’s state variables, such as $x_2$ and $x_4$, participate in the unstable modes. Note that $K_{P2}$, $K_{P4}$, $K_{I2}$, and $K_{I4}$ do not change the equilibrium point having just incidence in the eigenvalues pathway. In addition, a limit pathway of the eigenvalues has been observed at which the proportional and integral gains of the active and reactive power controllers do not change the system eigenvalues at all. For example, when $K_{P4} = 10$ p.u. and $K_{P4} = 500$ p.u., the same eigenvalues’ pathway is found. This limit pathway is located close to the base case pathway. This phenomenon has to be investigated in a future research. Note that the negative load model is a good approximation of this limit behavior.

With respect to the NL model, its validity for representing the power coming from the wind in power system stability analysis depends on the parameters of the active and reactive power controllers. For the base case, an NL model suffices, but when the controllers’ parameters are changed, especially those associated with the fast loop of the reactive power controller, an NL model gives a poor estimate of the system’s stability.

In summary,

a. Modal analysis is insensitive to $H_D$ variations (wind generator inertia).

b. Dominant eigenvalue pathways for the WTG and NL cases are similar when the base case parameters are used for the fast loop of the active and reactive power controllers. For any other value of these parameters, the validity of the NL model must be checked.
c. For the base case parameters of the active and reactive power controllers, the NL gives a good estimate of the system HB point.

d. When the parameters of the active and reactive power controllers are changed, the loading at the HB point does not change dramatically and the NL model makes a good estimation of it. However, the oscillation frequency may differ considerably depending on the controllers’ parameters.

2.8 Reduced-Order Model

The motivation for reducing the model order lies in the small influence of the WTG, Type-C WTG, over the system’s most dominant modes. As the generated power is the most influential variable over the system dynamics, a single equation model based on this power is obtained. Selective modal analysis (SMA) [44] is chosen as the reduction technique. More details of SMA are presented in the Appendix F.1.

Consider the system of Figure 2.23. For simplicity, replace the SG by a fixed voltage source such that $V_S = 1\angle 0^\circ$ p.u. Assume that the WTG is represented by its two-axis model. No pitch angle controller is considered. Thus, the state and algebraic variables of the system are

$$x = [E_{qD}', E_{dD}', \omega_r, x_1, x_2, x_3, x_4]^T$$

$$y = [V_{qr}, V_{dr}, I_{qr}, I_{dr}, P_{gen}, Q_{gen}, I_{ds}, I_{qs}, I_a, I_b, V_D, \theta_D, V_L, \theta_L]^T$$

$I_a$ and $I_b$ are the real and imaginary components of the current injected by the fixed voltage source ($I_S = I_a + jI_b$). All other variables have the same definition as in Section 2.1.

2.8.1 Case A

Consider base case parameters (shown in Appendix B) and assume that $v_{wind}^0 = 12$ m/s, $P_0^0 = 0.5$ p.u. and $Q_0^0 = 0.1$ p.u. The equilibrium point $(x_0, y_0)$ is found by setting the differential terms to zero and solving simul-
taneously the equations presented in Appendix C.

\[
E^r_{qD} = 0.9720 \text{ p.u.} \quad V_{qr} = -0.1939 \text{ p.u.} \quad I_{qs} = 0.2577 \text{ p.u.}
\]

\[
E^r_{dD} = 0.0048 \text{ p.u.} \quad V_{dr} = 0.0015 \text{ p.u.} \quad I_a = 0.2363 \text{ p.u.}
\]

\[
\omega_r = 451.4183 \text{ rad/s} \quad I_{qr} = 0.2508 \text{ p.u.} \quad I_b = -0.1451 \text{ p.u.}
\]

\[
x_1 = 0.2508 \text{ p.u.} \quad I_{dr} = 0.2863 \text{ p.u.} \quad V_D = 0.9689 \text{ p.u.}
\]

\[
x_2 = -0.1939 \text{ p.u.} \quad P_{gen} = 0.2980 \text{ p.u.} \quad \theta_D = -0.0370 \text{ rad}
\]

\[
x_3 = 0.2863 \text{ p.u.} \quad Q_{gen} = 0 \text{ p.u.} \quad V_L = 0.9013 \text{ p.u.}
\]

\[
x_4 = 0.0015 \text{ p.u.} \quad I_{ds} = 0.0095 \text{ p.u.} \quad \theta_L = -0.0828 \text{ rad}
\]

Use the wind speed as a system’s input and the generated active power as a system’s output. Thus, using the Jacobian matrix of the system set of DAEs at \((x^0, y^0)\), \(J\), the following linear model around the equilibrium point is obtained:

\[
\begin{bmatrix}
\Delta \dot{x} \\
0
\end{bmatrix}
= \begin{bmatrix}
A_s & B_s \\
C_s & D_s
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix}
+ \begin{bmatrix}
K \\
0
\end{bmatrix}
\Delta v_{wind}
\tag{2.133}
\]

\[
\begin{bmatrix}
\Delta P_{gen}
\end{bmatrix}
= \begin{bmatrix}
E_1 & E_2
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix}
\tag{2.134}
\]

where

\[
\Delta x = x - x^0
\tag{2.135}
\]

\[
\Delta y = y - y^0
\tag{2.136}
\]

\[
\Delta v_{wind} = v_{wind} - v^0_{wind}
\tag{2.137}
\]

\[
\Delta P_{gen} = P_{gen} - P^0_{gen}
\tag{2.138}
\]

\[
K = [0, 0, k_{\omega_r}, 0, 0, 0, 0]^T
\tag{2.139}
\]

\[
k_{\omega_r} = \frac{\omega_s}{2H_D} \frac{\partial T_m}{\partial v_{wind}}
\]

\[
= \frac{\rho \pi R^2 \omega_s^2}{4H_D S b \omega_r^0} v^0_{wind} 2 \left\{ 3C_p(\lambda, \theta) - v_{wind} \frac{\partial C_p}{\partial \lambda} \frac{\partial \lambda}{\partial \lambda} v_{wind} \right\}_0
\tag{2.140}
\]
\[ E_1 = \left[ \frac{\partial P_{\text{gen}}}{\partial E'_{qD}}, \frac{\partial P_{\text{gen}}}{\partial E'_{qD}}, 0, 0, 0, 0, 0 \right] \bigg|_{(x^0, y^0)} \]  
\[ E_2 = \left[ \frac{\partial P_{\text{gen}}}{\partial V_{qr}}, \frac{\partial P_{\text{gen}}}{\partial V_{dr}}, \frac{\partial P_{\text{gen}}}{\partial I_{qr}}, \frac{\partial P_{\text{gen}}}{\partial I_{dr}}, 0, 0, 0, 0, 0, 0 \right] \bigg|_{(x^0, y^0)} \]  
\[ E_1 = \left[ \frac{\partial P_{\text{gen}}}{\partial V_{qr}}, \frac{\partial P_{\text{gen}}}{\partial V_{dr}}, \frac{\partial P_{\text{gen}}}{\partial I_{qr}}, \frac{\partial P_{\text{gen}}}{\partial I_{dr}}, 0, 0, 0, 0, 0, 0 \right] \bigg|_{(x^0, y^0)} \]

Eliminate the algebraic variables to obtain

\[ \Delta \dot{x} = (A_s - B_s D_s^{-1} C_s) \Delta x + K \Delta v_{\text{wind}} \]  
\[ \Delta P_{\text{gen}} = (E_1 - E_2 D_s^{-1} C_s) \Delta x \]

At the equilibrium point, the matrix of participation factors, and eigenvalues of \( A_{\text{sys}} \) are

\[ P = \begin{bmatrix} 0.4984 & 0.4984 & 0.0018 & 0.0018 & 0 & 0.0008 & 0 \\ 0.4985 & 0.4985 & 0.0019 & 0.0019 & 0 & 0.0001 & 0.0006 \\ 0 & 0 & 0.0103 & 0.0103 & 0.9890 & 0.0030 & 0.0272 \\ 0.0007 & 0.0007 & 0.0277 & 0.0277 & 0.0106 & 0.0923 & 0.8165 \\ 0.0010 & 0.0010 & 0.4604 & 0.4604 & 0.0003 & 0.0123 & 0.0641 \\ 0.0005 & 0.0005 & 0.0244 & 0.0244 & 0 & 0.8787 & 0.0806 \\ 0.0010 & 0.0010 & 0.4735 & 0.4735 & 0 & 0.0129 & 0.0110 \end{bmatrix} \]

\[ \Lambda = \text{diag}\{\lambda_a, \lambda_b, \lambda_c, \lambda_d, \lambda_e, \lambda_f, \lambda_g\} \]

where

\[ \lambda_{a,b} = -2426.9 \pm j209.2 \quad \lambda_e = -0.0783 \]
\[ \lambda_{c,d} = -5.0698 \pm j0.2839 \quad \lambda_f = -2.3622 \]
\[ \lambda_g = -2.5385 \]

To apply SMA, a total number of \( h \) most relevant modes has to be identified, i.e., \( \lambda_i \forall i \in \{1, 2, ..., h\} \). In this case, \( h = 1 \) where the most dominant eigenvalue is \( \lambda_1 = \lambda_e = -0.0783 \) (observe the fifth column of matrix \( P \)). \( \Delta \omega_r \) has a high participation in this mode. The third component of the fifth column of \( P \) is related to \( \omega_r \). Note that this is defined by Equation (2.131), the vector of state variables. Although \( \Delta x_1 \) and \( \Delta x_2 \) also have some participation, this is neglected for now. Thus, the relevant eigenvalue is \( \lambda_1 \).
and the relevant state variable is $\Delta \omega_r$. Rearrange Equations (2.143) and (2.144) to obtain

\[
\begin{bmatrix}
\Delta \dot{\omega}_r \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
\Delta \omega_r \\
\dot{z}
\end{bmatrix} + \begin{bmatrix}
k_{\omega_r} \\
0
\end{bmatrix} \Delta v_{wind}
\] (2.147)

\[
\begin{bmatrix}
\Delta P_{gen}
\end{bmatrix} = \begin{bmatrix}
\tilde{E}_1 & \tilde{E}_2
\end{bmatrix} \begin{bmatrix}
\Delta \omega_r \\
\dot{z}
\end{bmatrix}
\] (2.148)

where $z = [\Delta E'_{qD}, \Delta E'_{dD}, \Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T$ is the vector of irrelevant variables. Then, considering $z(t) = (\lambda_1 I - A_{22})^{-1} A_{21} \Delta \omega_r(t)$, reduce the model order as

\[
\Delta \dot{\omega}_r = \alpha_{\omega_r} \Delta \omega_r + k_{\omega_r} \Delta v_{wind}
\] (2.149)

\[
\Delta P_{gen} = \alpha_P \Delta \omega_r
\] (2.150)

where $\alpha_{\omega_r} = A_{11} + A_{12} (\lambda_1 I - A_{22})^{-1} A_{21}$ and $\alpha_P = \tilde{E}_1 + \tilde{E}_2 (\lambda_1 I - A_{22})^{-1} A_{21}$.

Thus,

\[
\Delta \dot{P}_{gen} = \alpha_P \Delta \dot{\omega}_r = \alpha_P \alpha_{\omega_r} \Delta \omega_r + \alpha_P k_{\omega_r} \Delta v_{wind}
\] (2.151)

\[
\Rightarrow \dot{P}_{gen} = \Delta \dot{P}_{gen} = \alpha_{\omega_r} (P_{gen} - P_{gen}^0) + \alpha_P k_{\omega_r} (v_{wind} - v_{wind}^0)
\] (2.152)

Finally,

\[
\dot{P}_{gen} = \beta_1 P_{gen} + \beta_2 v_{wind} + \beta_3
\] (2.153)

where

\[
\beta_1 = \alpha_{\omega_r}
\] (2.154)

\[
\beta_2 = \alpha_P k_{\omega_r}
\] (2.155)

\[
\beta_3 = -\alpha_{\omega_r} P_{gen}^0 - \alpha_P k_{\omega_r} v_{wind}^0
\] (2.156)

Numerically,

\[
\alpha_{\omega_r} = \lambda_1 = -0.0783 \quad \beta_1 = -0.0783
\]

\[
\alpha_P = 0.0020 \quad \Rightarrow \beta_2 = +0.0058
\]

\[
k_{\omega_r} = 2.9177 \quad \beta_3 = -0.0467
\] (2.157)

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Note that at steady state, the power output is defined by

\[
P_{\text{gen}} = -\frac{\beta_2}{\beta_1} v_{\text{wind}} - \frac{\beta_3}{\beta_1}
\]

(2.158)

If wind speed decreases to 11.5 m/s, \( P_{\text{gen}} = 0.2607 \) p.u. When the full model is used, the total power injected by the WTG is 0.2614 p.u.

To verify the validity of this reduced-order model in time domain simulations, consider the original system in which the dynamic model of the SG is incorporated (see Appendix C) and consider the wind speed profile shown in Figure 2.31. In addition, this figure also shows the evolution of \( P_{\text{gen}} \) and \( \omega_r \) when the full order model is used. The wind speed, power and angular speed are per unitized using their initial values. The comparison between the two-axis and first-order models is presented in Figure 2.32. A remarkable agreement is observed. Although the figure only shows the comparison in terms of \( P_{\text{gen}} \) and \( P_m \), a similar noticeable agreement is found when the other state and algebraic variables are compared. In addition, two other cases are considered. In the first one, the inertia of the WTG is reduced to 0.5 s (see Figures 2.33 and 2.34) and in the second one a varying load is considered (see Figure 2.35). As shown, the first-order model can mimic very well the impact of the WTG over the system dynamics. This is true for many analyzed cases except when some power controller parameters are changed. This case is considered next.

2.8.2 Case B

The WTG may have more impact over the system dynamics depending on the fast loop parameters of the power controllers (see Section 2.7.1). For example, a proper selection of gains \( K_{I4} \) and \( K_{P4} \) can reduce oscillations of the dominant modes around the HB point. In fact, a few WTG’s variables participate in the unstable modes when such gains are used.

In the previous case, where base case parameters are considered, a first-order model is enough to represent the WTG in the power system. When the fast loop’s parameters of the power controllers are modified, a first-order model fails to give correct results. Moreover, although for the base case the system dynamics are insensitive to inertia, the inertia may alter the impact of the fast loop parameters. For simplicity, consider the fast loop
Figure 2.31: Evolution of $v_{\text{wind}}$ [p.u.], $\omega_r$ [p.u.] and $P_{\text{gen}}$ [p.u.] when $H_D = 4$ s. $V_{\text{wind,base}} = 12$ m/s, $\omega_{r,\text{base}} = 451.4183$ rad/s², $P_{\text{gen,base}} = 0.2980$ p.u.

Figure 2.32: Comparison of $P_{\text{gen}}$ and $P_m$ by using the two-axis and first-order order model for the WTG with $H_D = 4$ s.
Figure 2.33: Evolution of $v_{\text{wind}}$ [p.u.], $\omega_r$ [p.u.] and $P_{\text{gen}}$ [p.u.] when $H_D = 0.5$ s. $V_{\text{wind,base}} = 12$ m/s, $\omega_{r,\text{base}} = 451.4183$ rad/s$^2$, $P_{\text{gen,base}} = 0.2980$ p.u.

Figure 2.34: Comparison of $P_{\text{gen}}$ and $P_m$ by using the two-axis and first-order model for the WTG with $H_D = 0.5$ s.
gains of the speed controller. The ratio $\frac{K_{I2}}{K_{P2}} = 5$ is kept constant. The parameter $K_{I2}$ is varied from 0.1 to 5 p.u. and the WTG inertia is assumed to be 0.5 s. When the first-order model is used, the model’s parameters have discontinuities when $K_{I2} \in \{0.16, 1.18, 2.76\}$ (see Figure 2.36). At very low gains, this discontinuity is evidence of an abrupt change in the modal structure which leads to a poor representation of the WTG by the first-order model. This phenomenon can be explained by estimating the number of relevant modes. By using participation factors, the modes in which $\omega_r$ (variable of interest) has a participation above 30% are selected as relevant modes. Say that $h$ modes are selected. Then, $h$ variables with the highest participation on these modes are selected. In Figure 2.37, the number of relevant modes is presented. This is consistent with the discontinuities of the first-order model’s parameters as the number of relevant modes changes when $K_{I2} \in \{0.16, 1.18, 2.76\}$.

Finally, a particular case is considered. Follow the same procedure as in Case A with the same parameters but $K_{I2} = 0.5$ p.u., $K_{P2} = 0.1$ p.u. and $H_D = 0.5$ s. The following participation factor matrix and eigenvalues are
Figure 2.36: Parameters of the 1st order model when $K_{I2}$ is varied from 0.1 to 5 p.u.

Figure 2.37: Number of relevant modes when $K_{I2}$ is varied from 0.1 to 5 p.u.
\[ P = \begin{bmatrix}
0.9921 & 0.0048 & 0.0004 & 0.0034 & 0.0008 & 0 & 0 \\
0.0049 & 0.9447 & 0.0484 & 0.0005 & 0 & 0.0001 & 0.0001 \\
0 & 0.0190 & 0.4344 & 0.0170 & 0.0034 & 0.4425 & 0.4425 \\
0 & 0.0109 & 0.1335 & 0.0002 & 0.0016 & 0.3234 & 0.3234 \\
0 & 0.0205 & 0.3602 & 0.0115 & 0.0013 & 0.2340 & 0.2340 \\
0.0010 & 0 & 0.0058 & 0.0182 & 0.9851 & 0 & 0 \\
0.0020 & 0.0001 & 0.0172 & 0.9493 & 0.0078 & 0 & 0 \\
\end{bmatrix} \] (2.159)

\[ \Lambda = \text{diag}\{\lambda_a, \lambda_b, \lambda_c, \lambda_d, \lambda_e, \lambda_f, \lambda_g\} \] (2.160)

where

\[ \lambda_a = -2415.4 \quad \lambda_d = -5.1099 \]
\[ \lambda_b = -240.9 \quad \lambda_e = -2.3701 \]
\[ \lambda_c = -11.4191 \quad \lambda_{f,g} = -0.7311 \pm j0.4514 \]

Note that the variable of interest, \( \omega_r \), has a participation above than 40\% in modes \( \lambda_c \) (third column of matrix \( P \)), \( \lambda_f \) (sixth column) and \( \lambda_g \) (seventh column). By observing the participation factors in these three columns, \( \omega_r \), \( x_1 \) and \( x_2 \) are selected as relevant variables. Thus, the reduction is performed considering these three relevant modes and three relevant variables. After eliminating the algebraic variables, re-order the linearized system of equations such that

\[
\begin{bmatrix}
\dot{r} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
r \\
z
\end{bmatrix} +
\begin{bmatrix}
K' \\
0
\end{bmatrix}
\Delta v_{\text{wind}}
\] (2.161)

\[
\Delta P_{\text{gen}} =
\begin{bmatrix}
\tilde{E}_1 & \tilde{E}_2
\end{bmatrix}
\begin{bmatrix}
r \\
z
\end{bmatrix}
\] (2.162)

where \( K = [k_{\omega_r}, 0, 0]^T \), \( r = [\Delta \omega_r, \Delta x_1, \Delta x_3]^T \) which is the vector of relevant variables and \( z = [\Delta E'_{qD}, \Delta E'_{dD}, \Delta x_3, \Delta x_4]^T \) which is the vector of irrelevant variables. Then, reduce the model order considering two constant matrices \( M_0 \) and \( K_0 \) to obtain

\[
\dot{r} = (A_{11} + M_0) r + k_{\omega_r} \Delta v_{\text{wind}}
\] (2.163)

\[
\Delta P_{\text{gen}} = \left( \tilde{E}_1 + \tilde{E}_2 K_0 \right) r
\] (2.164)
$M_0$ and $K_0$ are calculated by solving the following equations (see Appendix F.1 for more details):

\[
M_0 = [H(\lambda_1)v_1, H(\lambda_2)v_2, H(\lambda_3)v_3][v_1, v_2, v_3]^{-1} \quad (2.165)
\]
\[
K_0 = [K(\lambda_1)v_1, K(\lambda_2)v_2, K(\lambda_3)v_3][v_1, v_2, v_3]^{-1} \quad (2.166)
\]

where $v_1$, $v_2$ and $v_3$ correspond to the right eigenvectors of the relevant modes $\lambda_1$, $\lambda_2$ and $\lambda_3$, respectively. Note that $v_i \in \mathbb{C}^{3 \times 1}$, $\forall \ i = \{1, 2, 3\}$ as they just consider the entries associated to the relevant variables. The functions $H(\lambda)$ and $K(\lambda)$ are defined as

\[
H(\lambda) = A_{12} (\lambda I - A_{22})^{-1} A_{21} \quad (2.167)
\]
\[
K(\lambda) = (\lambda I - A_{22})^{-1} A_{21} \quad (2.168)
\]

Numerically, the following third-order model is obtained in terms of the original variables:

\[
\begin{bmatrix}
\dot{\omega}_r \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
\omega_r \\
x_1 \\
x_2
\end{bmatrix} + \beta_2 v_{wind} + \beta_3
\quad (2.169)
\]
\[
P_{gen} = \beta_4 \begin{bmatrix}
\omega_r \\
x_1 \\
x_2
\end{bmatrix} + \beta_5
\quad (2.170)
\]

where

\[
\beta_1 = A_{11} + M_0 = \begin{bmatrix}
-4.6769 & -175.6340 & -1728.5 \\
-0.0641 & -2.7688 & -27.2906 \\
-0.0134 & -0.0464 & -5.4232
\end{bmatrix} \quad (2.171)
\]
\[
\beta_2 = K' = \begin{bmatrix}
23.3415 \\
0 \\
0
\end{bmatrix} \quad (2.172)
\]
\[
\beta_3 = - (A_{11} + M_0) \begin{bmatrix}
\omega_r^0 \\
x_1^0 \\
x_2^0
\end{bmatrix} - K' v_{wind}^0 = \begin{bmatrix}
1562.1 \\
24.7 \\
5.1
\end{bmatrix} \quad (2.173)
\]
\[
\beta_4 = \tilde{E}_1 + \tilde{E}_2 K_0 = \begin{bmatrix} 0.0148 & 0.5538 & 5.4581 \end{bmatrix}
\]

(2.174)

\[
\beta_5 = \left( \tilde{E}_1 + \tilde{E}_2 K_0 \right) \begin{bmatrix} \omega_r^0 \\ x_1^0 \\ x_2^0 \end{bmatrix} = -5.5332
\]

(2.175)

In Figures 2.38 and 2.39, a comparison of \(T_m\), \(P_{\text{gen}}\) and \(E'_q\) is presented when the two-axis, the first-order and third-order models are used. Note that as this case has three relevant modes, the third-order model rather than the first-order model represents properly the WTG in the power system. The procedure and simulation is performed again with higher inertia (see Figure 2.40). Observe that as the inertia is higher the number of relevant modes tends to 1. In other words, with a high inertia, say an inertia greater than 2 s, the first-order model can represent properly the WTG in the power system. According to the literature, a typical WTG has a DFIG inertia of about 0.5 – 0.9 s and a turbine inertia of about 2.5 – 4.5 s in the WTG power base [45, 46]. This implies that the total inertia is in the range of 3 – 5.4 s. It seems that the issue of having a low inertia, which can jeopardize the WTG representation by a first-order model, is not realistic. However, when

![Graph showing the dynamic evolution of \(T_m\) and \(P_{\text{gen}}\) with different models.](image)

Figure 2.38: \(T_m\) and \(P_{\text{gen}}\) dynamic evolution when \(K_{I2} = 0.5\) p.u.
Figure 2.39: $E_q$ dynamic evolution when $K_{I2} = 0.5$ p.u.

Figure 2.40: Number of relevant modes when $K_{I2}$ and $H_D$ are varied from 0.1 to 5 p.u. and from 0.5 to 3 s, respectively.
the inertia is per unitized using the system’s power base, it may become very low. Assuming that both the WTG and system voltage bases are equal, then the inertia in the system base is calculated as

$$H_D = H_D^{WTG} \frac{S_{WTG}}{S_{base}}$$  \hspace{1cm} (2.176)

where $H_D^{WTG}$ is the inertia in WTG base, $S_{WTG}$ is the nominal power of the WTG and $S_{base}$ is the system power base.

A similar analysis is performed when the gains of the fast loop of the reactive power controller are varied. Keep constant the ratio $\frac{K_{I4}}{K_{P4}} = 5$ and vary $K_{I4}$ from 0.1 to 5 p.u. Along this range of parameters, there exists just one relevant mode. Thus, the first-order model is appropriate to mimic the behavior of the WTG. This is supported by the simple fact that the reactive power is not strongly coupled with the active power and thus not strongly coupled with the electrical rotor speed—the most relevant variable.
Wind farms contain many wind turbines and their detailed modeling may be unaffordable due to computational burden. For example, Roscoe Wind Farm, Texas, one of the largest wind farms in the world, has an installed capacity of 780 MW, and 627 wind turbines [47]. In order to reduce the dimensionality, aggregation techniques are used to obtain equivalent models.

In a wind farm, the power output may vary from zero to the installed capacity. The equivalent model should be valid for a wide operation range. Moreover, the equivalent model should be accurate enough during abnormal operation such as faults, wind speed variations or voltage sags/swells [48]. A proper equivalent model can be easily obtained for fixed-speed wind turbines where a one-to-one correspondence between wind speed and active power output exists. In this case, aggregation is performed by adding the mechanical power of each wind turbine and by using an equivalent induction generator which receives the total mechanical power. Parameters of the equivalent generator are the same as those of the individual generators [48–50]. Also, weighted least square techniques have been used to estimate an equivalent model by considering as main variables the active and reactive power as well as the voltage magnitude at the point of common coupling (PCC) [48]—point of connection of the wind farm to the power system. In general, it has been observed that these models are sufficiently accurate even during faults due to the lack of interaction between individual turbines [50]. In the case of variable-speed wind turbines, it has been seen that mutual interaction exists between turbines. If it is negligible, then aggregation is possible and the wind farm can be represented by a single turbine-generator. However, in order to investigate mutual interaction (especially interaction of back-to-back converters), detailed wind farm models are required. It is warned that model oversimplification can lead to a loss of accuracy. Converter blocking protection may be altered and torsional oscillations may be incorrectly predicted.
when an inappropriate aggregation is performed [51].

Other important issues in aggregation are the wind speed distribution and wind farm layout which depend on the wind farm location. In the case of wind farms located in flat valleys or offshore, the wind speed can be considered homogenous and the turbines are placed in rows and columns equally separated. If the incoming wind is parallel to the rows, turbines located in the same column will receive the same wind speed and they can be represented by an equivalent turbine-generator. Turbines located in different columns would have a different incoming wind speed due to the wake effect [49]. Thus, the wind farm aggregated model will consist of as many single WTGs as the number of columns. If the wake effect is negligible, all turbines would receive the same wind speed and just one equivalent turbine-generator can represent the whole wind farm. In the case that the wind direction changes, an equivalent WTG is obtained for every group of turbines receiving the same wind speed [49]. Most of the aggregation techniques are based on adding turbines with identical incoming winds. When the wind farm is not located in smooth areas, turbines are placed irregularly throughout the farm and the spacing between turbines is greater than in the previous case. Turbines with identical wind speed are rarely found and most of the aggregation techniques cannot be applied directly [52–54].

The turbine location clearly affects the wind that every WTG faces and the way that any aggregation technique should be performed. Turbines are typically separated by at least 5 times their turbine diameter to reduce wake loss. Wake refers to the reduction of the wind speed when an air mass passes through a turbine’s blades. In [55], a wake model and a procedure to estimate the wind speed at every WTG are presented. Note that wind direction is another important factor as it defines the number of shadowing turbines that reduce the incoming wind speed of a downstream WTG. The higher the number of shadowing turbines, the lower the incoming wind speed in a downstream turbine. With respect to wind speed modeling, turbulence and wind gust have also been considered. A wind model of four components is generally considered [8] as

\[ v_{\text{wind}}(t) = v_{\text{wind},a} + v_{\text{wind},r}(t) + v_{\text{wind},g}(t) + v_{\text{wind},t}(t) \]  \hspace{1cm} (3.1)

where \( v_{\text{wind},a} \) is the average wind speed, \( v_{\text{wind},r} \) is the ramp wind component,
\( v_{\text{wind},g} \) is the gust component and \( v_{\text{wind},t} \) is the turbulence component. The last component is typically modeled by stochastic functions which are not considered in this thesis. The gust and ramp component are combined and defined as a variable wind component (it is also called gust). Therefore, an average value and a variable component are used to represent the wind speed profile at every WTG. For simplicity, the wake effect is represented as a fixed percentage reduction of the wind speed when there exists a shadowing turbine upstream. A more elaborate model should be considered in future research.

### 3.1 Aggregated Model

#### 3.1.1 Proposed Model

In a wind farm, the total mechanical power is

\[
P_{m}^{e} = \sum_{i=1}^{n_{g}} P_{m}^{i} = \sum_{i=1}^{n_{g}} \frac{1}{2} \rho A_{wt} C_{p}^{i}(\lambda^{i}, \theta^{i}) v_{\text{wind}}^{i} \tag{3.2}
\]

where \( n_{g} \) is the number of WTGs in the wind farm. In order to obtain an equivalent or aggregated model, this total power is defined as the mechanical power applied to the shaft of the equivalent generator. In general, the aggregation technique is based on the idea of adding the power of the individual WTGs.

Consider that all WTGs have identical parameters and they do not necessarily operate at the same wind speed. As the speed control maximizes the power extraction from the wind, the power coefficient is also maximized. At steady state, the power coefficient is maximum at every WTG regardless of the wind speed. Thus, \( \forall \ i, \ C_{p}^{i}(\lambda^{i}, \theta^{i}) = C_{p}^{\text{max}} \) where it is also assumed that \( \theta^{i} = 0 \) (wind speed remains within its limits and no pitch controller is required). Using the definition of the power coefficient, Equation (2.3), it can be proved that \( C_{p}^{\text{max}} = 0.4382 \). Then,

\[
P_{m}^{e} = \frac{1}{2} \rho A_{wt} C_{p}^{\text{max}} \sum_{i=1}^{n_{g}} v_{\text{wind}}^{i} \tag{3.3}
\]
Consider that the equivalent wind turbine has the same blade length as the individual turbines. Also, assume that there exists an equivalent active power controller such that in steady state $C_p^e(\lambda^e, \theta^e) = C_p^{\text{max}}$. Then the equivalent wind power becomes

$$P_m^e = \frac{1}{2} \rho A_{wt} C_p^e(\lambda^e, \theta^e) \sum_{i=1}^{ng} v_{\text{wind}}^i$$

(3.4)

Define an equivalent wind speed as

$$v_{\text{wind}}^e = \left( \frac{1}{ng} \sum_{i=1}^{ng} v_{\text{wind}}^i \right)^{\frac{3}{2}}$$

(3.5)

Thus,

$$P_m^e = ng \left[ \frac{1}{2} \rho A_{wt} C_p^e(\lambda^e, \theta^e) v_{\text{wind}}^e \right]$$

(3.6)

In addition, consider that the equivalent angular speed, $\omega_r^e$, is defined in the same speed range as the angular speed of the individual turbines, e.g., $\omega_r^e \in [0.7 \times \omega_s, 1.2 \times \omega_s]$. Then, the torque equation is

$$T_m^e = ng \frac{1}{2} \rho \pi R^2 \omega_s C_p^e(\lambda^e, \theta^e) \frac{v_{\text{wind}}^e}{\omega_r^e} \left[ \text{p.u.} \right]$$

$$= B^e \omega_s C_p^e(\lambda^e, \theta^e) \frac{v_{\text{wind}}^e}{\omega_r^e} \left[ \text{p.u.} \right]$$

(3.7)

where $B^e = ng \times B$.

So far, using an equivalent wind speed, the mechanical power and torque of the aggregated model have been obtained. To determine all the aggregated model parameters, proceed in the same fashion as before, i.e., consider that the equivalent power is the sum of the individual WTG’s power. Then, define equivalent variables and parameters that keep the equivalency. The chosen aggregation procedure is similar to the one used for fixed-speed WTGs [54]. The equivalent reference for active power and reactive power correspond to
the sum of references over all WTGs as

\[ P_{\text{ref}}^e = \sum_{k=1}^{ng} P^e_{\text{ref},k} = C \sum_{k=1}^{ng} \omega^e_r = C^e \omega^e_r, \quad (3.8) \]

\[ Q_{\text{ref}}^e = \sum_{k=1}^{ng} Q^e_{\text{ref},k} = \sum_{k=1}^{ng} Q_{\text{ref}} = ng \times Q_{\text{ref}}, \quad (3.9) \]

where \( C^e = ng \times C \) is the equivalent coefficient specifying the equivalent power reference. Now, focussing on the equation of motion, while \( T_m^e \) is defined by Equation (3.7), \( H_D^e \) and \( T_e^e \) are defined by

\[ H_D^e = \sum_{k=1}^{ng} H_{Dk} \quad (3.10) \]

\[ T_e^e = \sum_{k=1}^{ng} T_{ek} = E'_{qD}^e \sum_{k=1}^{ng} I_{qsk} + E'_{dD}^e \sum_{k=1}^{ng} I_{dsk} = E'_{qD}^e I_{qs}^e + E'_{dD}^e I_{ds}^e \quad (3.11) \]

where the equivalent stator currents are

\[ I_{qs}^e = \sum_{k=1}^{ng} I_{qsk} \quad I_{ds}^e = \sum_{k=1}^{ng} I_{dsk} \quad (3.12) \]

In the equivalent model, voltages are of the same order of magnitude as in the individual WTGs and currents are about \( ng \) times larger. The total power injected to the rotor circuit is

\[ P_r^e = V_{qr}^e \sum_{k=1}^{ng} I_{qrk} + V_{dr}^e \sum_{k=1}^{ng} I_{drk} = V_{qr}^e I_{qr}^e + V_{dr}^e I_{dr}^e \quad (3.13) \]

where the equivalent rotor currents are

\[ I_{qr}^e = \sum_{k=1}^{ng} I_{qrk} \quad I_{dr}^e = \sum_{k=1}^{ng} I_{drk} \quad (3.14) \]

Now, take a close look at the equations related to the equivalent con-
trollers:
\[
\frac{dx_e^c}{dt} = K_{I2} \left[ K_{P1} \left( P_{ref}^e - P_{gen}^e \right) + x_1^e - I_{q2}^e \right] \quad (3.15)
\]
\[
\frac{dx_e^4}{dt} = K_{I4} \left[ K_{P3} \left( Q_{ref}^e - Q_{gen}^e \right) + x_3^e - I_{dr}^e \right] \quad (3.16)
\]
\[
0 = -V_{qr}^e + K_{P2}^e \left[ K_{P1} \left( P_{ref}^e - P_{gen}^e \right) + x_1^e - I_{q2}^e \right] + x_2^e \quad (3.17)
\]
\[
0 = -V_{dr}^e + K_{P4}^e \left[ K_{P3} \left( Q_{ref}^e - Q_{gen}^e \right) + x_3^e - I_{dr}^e \right] + x_4^e \quad (3.18)
\]

Here, \( P_{ref}^e, P_{gen}^e, x_1^e, I_{q2}^e, Q_{ref}^e, Q_{gen}^e, x_3^e, I_{dr}^e \) are magnified \( ng \) times by the aggregation; thus,
\[
K_{P2}^e = \frac{K_{P2}}{ng} \quad K_{I2}^e = \frac{K_{I2}}{ng} \quad (3.19)
\]
\[
K_{P4}^e = \frac{K_{P4}}{ng} \quad K_{I4}^e = \frac{K_{I4}}{ng} \quad (3.20)
\]

Note that in steady state, \( x_2^e = V_{q2}^e \) and \( x_4^e = V_{dr}^e \) which are of the same order of magnitude as the individual WTGs.

Finally, by inspection of the DAEs, it is required that equivalent electrical parameters are calculated as if all WTGs are in parallel (equivalent resistances and reactances). To consider the wind farm network, obtain the wind farm equivalent impedance, \( Z_{equiv} \), at the PCC when every WTG is short-circuited. Then, the equivalent WTG is connected through an equivalent line to the grid. The equivalent line has a series impedance equal to \( Z_{equiv} \).

In summary, the scaling of the following parameters is needed:
\[
X_m^e = \frac{X_m}{ng} \quad B^e = ng \times B
\]
\[
X_s^e = \frac{X_s}{ng} \quad C^e = ng \times C
\]
\[
X_r^e = \frac{X_r}{ng} \quad Q_{ref}^e = ng \times Q_{ref}
\]
\[
X_s' = \frac{X_s'}{ng} \quad K_{P2}^e = \frac{K_{P2}}{ng}
\]
\[
R_s = \frac{R_s}{ng} \quad K_{P4}^e = \frac{K_{P4}}{ng}
\]
\[
R_r = \frac{R_r}{ng} \quad K_{I2}^e = \frac{K_{I2}}{ng}
\]
\[
H_D^e = ng \times H_D \quad K_{I4}^e = \frac{K_{I4}}{ng}
\]

All other parameters remain equal to those used for the individual turbines \( (K_{I1}, K_{P1}, T_0', \text{ others}) \).
3.1.2 Slootweg’s Method

This method is based basically on (a) the linearization of the active power reference as a function of the electrical rotor speed and (b) the retention of the equation of motion of every turbine neglecting any other dynamics related to the controllers and flux linkages. Then, the total electrical power injected by a wind farm to the grid is calculated by summing over the electrical power of every turbine. The following steps are considered [9, 10, 56]:

a. Assume that the wind speed forecast at every turbine location is given.

b. Assume that the rotor speed control is ideal and it is always possible to operate at the optimal power point, i.e., \( \forall \ i \in \{1, 2, ..., n\} \), \( C_p(\lambda, \theta)^i = C_p^{\text{max} i} \). Here, \( n \) is the total number of turbines in the wind farm.

c. Using the forecasted wind speed, calculate the mechanical power for every turbine, i.e., \( \forall \ i \in \{1, 2, ..., n\} \), \( P_{w}^i = \frac{1}{2} \rho A_{\text{wt}} C_p^{\text{max} i} v_{\text{wind}}^i r_{\text{in}}^3 \).

d. For every turbine-generator, linearize the power-speed characteristic for control (Figure 2.4), i.e., \( \forall \ i \in \{1, 2, ..., n\} \), \( P_e^i = K^i (\omega_r^i - \omega_{\text{min}}^i) \). Here, \( K^i \) and \( \omega_{\text{min}}^i \) are fixed constants. Then,

\[
P_e^i = K^i (\omega_r^i - \omega_{\text{min}}^i) \Rightarrow \frac{d\omega_r^i}{dt} = \frac{1}{K^i} \frac{dP_e^i}{dt}
\]  \( (3.21) \)

Thus, the equation of motion becomes

\[
\frac{2H_D^i}{\omega_s} \frac{d\omega_r^i}{dt} = P_w^i - P_e^i \quad (3.22)
\]

\[
\Rightarrow \frac{dP_e^i}{dt} = \omega_s K^i \frac{P_w^i - P_e^i}{2H_D^i} \quad (3.23)
\]

Note that Slootweg defined the equation of motion in terms of power. In this work, the equation of motion is defined in terms of torque.

e. For every turbine-generator, integrate the equation of motion to obtain \( \omega_r^i \) and \( P_e^i \).

f. Add the electrical power of every turbine-generator \( P_e^i \) and inject it to the point of common coupling (PCC, point of connection of the wind farm and the electric grid).
3.2 Four-Bus Test System

Consider the 4-bus test system of Figure 2.23. Assume that instead of a unique turbine there are 10 WTGs that are directly connected to the bus with no transformer. Thus, no wind farm network is considered. After the WTGs are aggregated, the equivalent model is reduced by using SMA. Two cases are presented. In the first one, the WTG’s parameters are such that only one relevant mode exists. In the second case, more than one relevant mode exists. In addition, this procedure is compared with Slootweg’s method. Data of the synchronous generator, lines, transformer and load is obtained from Appendix B. Consider the following parameters for every WTG:

\[ X_s = 35.5470 \text{ [p.u.] } \quad R_s = 0.1015 \text{ [p.u.] } \]

\[ X_r = 35.8590 \text{ [p.u.] } \quad R_r = 0.0880 \text{ [p.u.] } \]

\[ X_m = 35.0920 \text{ [p.u.] } \quad B = 4.78 \times 10^{-5} \text{ [pu s}^3/\text{m}^3] \]

\[ X'_s = 1.2056 \text{ [p.u.] } \quad C = 3.8326 \times 10^{-10} \text{ [pu s}^3] \]

\[ T_0 = 1.0809 \text{ [s]} \quad D = 0.1667 \]

\[ K_{P1} = 1 \text{ [p.u.]} \quad K_{I1} = 5 \text{ [p.u.] } \]

\[ K_{P3} = 1 \text{ [p.u.]} \quad K_{I3} = 5 \text{ [p.u.] } \]

\[ K_{P4} = 10 \text{ [p.u.]} \quad K_{I4} = 5 \text{ [p.u.] } \]

Assume that the wind speed decreases by 2% after the wind passes through a turbine’s blades. Also, consider that the WTGs are aligned with the wind direction. Thus, if the free incoming wind speed is 12 m/s, then the wind speed at every turbine is listed in Table 3.1.

<table>
<thead>
<tr>
<th>WTG No.</th>
<th>( v_{\text{wind}} ) [m/s]</th>
<th>WTG No.</th>
<th>( v_{\text{wind}} ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.0000</td>
<td>6</td>
<td>10.8470</td>
</tr>
<tr>
<td>2</td>
<td>11.7600</td>
<td>7</td>
<td>10.6301</td>
</tr>
<tr>
<td>3</td>
<td>11.5248</td>
<td>8</td>
<td>10.4175</td>
</tr>
<tr>
<td>4</td>
<td>11.2943</td>
<td>9</td>
<td>10.2092</td>
</tr>
<tr>
<td>5</td>
<td>11.0684</td>
<td>10</td>
<td>10.0050</td>
</tr>
</tbody>
</table>
3.2.1 Case A

In addition to the WTG’s parameters presented, consider that $K_{P2} = 10$ p.u., $K_{P4} = 10$ p.u., $K_{I2} = 5$ p.u. and $H_D = 0.2$ s. For dynamic simulations, from $t = 0$ to $t = 20$ s, constant wind speeds are assumed (see Table 3.1). At $t = 20$ s, a wind gust is applied which travels along the WTGs. Thus, the new wind speed at every WTG is given by $v_{\text{wind}}^{\text{new}} = v_{\text{wind}} + v_{\text{gust}}$. After the wind gust passes through a turbine’s blades, the gust’s wind speed is reduced 2%. Also, a traveling time of 2 s between WTGs is considered. The wind gust causes wind speed variations of about $\pm 1.5$ m/s around the initial speed. Using these parameters and Equation (3.5), the wind speed profile at every WTG, and the equivalent speed are calculated (see Figure 3.1).

In Figure 3.2, the power generated by every WTG is presented. Then, the WTGs are aggregated using the proposed aggregation technique. Furthermore, the aggregated model is reduced using a first-order model. The parameters of the first-order model are

$$
\beta_1 = -0.1761 \quad (3.24)
$$

$$
\beta_2 = +0.0134 \quad (3.25)
$$

$$
\beta_3 = -0.0979 \quad (3.26)
$$

Finally, these methods are compared with Slootweg’s method. In Figure 3.3, the total power generated by the wind farm, using all the mentioned method, is presented. A remarkable agreement is observed. Observe that Slootweg’s method generates an offset in the variables which is due basically to the linearization of the power reference. The proposed method does not show that problem as the aggregated and reduced-order model are obtained from the same initial condition of the full order model. Still, the dynamics are very similar in all cases (see Figure 3.4).

3.2.2 Case B

Consider that $K_{P2} = 1$ p.u., $K_{P4} = 10$ p.u., $K_{I2} = 0.5$ p.u. and $H_D = 0.05$ s. The same wind profile as Case A is considered. In Figure 3.5, the total power generated by the WTGs, and the synchronous generator torque are presented. The two-axis and the aggregated model are used. Observe that
Figure 3.1: Wind speed at every WTG and equivalent speed (4-bus test system).

Figure 3.2: Power generated by every WTG (4-bus test system).
Figure 3.3: Total power generated by the WTGs ($P_{\text{gen}}^\text{total}$) and mechanical torque of the SG (4-bus test system).

Figure 3.4: Synchronous generator’s field voltage (4-bus test system).
in this case the proposed aggregated model, due to averaging of the wind speed, is not able to follow the behavior of the full order model as in Case A. Another option that can be explored is to represent every WTG by its reduced-order model. In the case of a first-order model, the aggregation can be performed in a straightforward way by summing over the individual powers and finding equivalent parameters. In the case of a third-order model, the aggregation will present the same problem of the proposed aggregation method because of the need for finding an average electrical rotor speed. When the aggregated model is reduced, three relevant modes are found. Thus, while the first-order model poorly represents the aggregated model, the third-order model presents very good results. The reduced-order models are defined by Equations (2.153) and (2.169)-(2.170). The first-order model parameters are

\[
\begin{align*}
\beta_1 &= -6.0611 \\
\beta_2 &= +0.4591 \\
\beta_3 &= -3.3604
\end{align*}
\] (3.27) (3.28) (3.29)
The third-order model parameters are

\[
\beta_1 = \begin{bmatrix}
-5.1329 & -174.6996 & -1774.5482 \\
-0.0650 & -2.5496 & -25.9303 \\
-0.0136 & -0.0065 & -5.1530 \\
\end{bmatrix}
\] (3.30)

\[
\beta_2 = \begin{bmatrix}
25.9167 \\
0.0000 \\
0.0000 \\
\end{bmatrix}
\] (3.31)

\[
\beta_3 = \begin{bmatrix}
1716.8097 \\
25.0767 \\
5.1373 \\
\end{bmatrix}
\] (3.32)

\[
\beta_4 = \begin{bmatrix}
0.0150 & 0.5099 & 5.1861 \\
\end{bmatrix}
\] (3.33)

\[
\beta_5 = -5.5749
\] (3.34)

While the first-order model cannot attain better results, the third-order model may mimic more properly the behavior of the full-order model by improving the aggregation technique. Also, as mentioned before, the aggregation stage may even be avoided by representing every single WTG by its third-order model. In Figures 3.6 and 3.7, by using the reduced-order models and Slootweg’s method, the total power generated by the WTGs, and the synchronous generator torque are presented. Observe that while the first-order model fails to follow the full-order model, the third-order model and Slootweg’s method give good results. Slootweg’s method causes an offset, which is its main disadvantage. From observing Figure 3.7, it seems that Slootweg’s method has also the disadvantage of producing a smoother \( P_{gen}^e \) than the full-order and third-order models.
Figure 3.6: $P_{gen}$ and $T_m$ by using the reduced-order models and Slootweg’s Method (4-bus test system).

Figure 3.7: Zoom in of Figure 3.6 (4-bus test system).
CHAPTER 4

POWER SYSTEM STABILITY

A power system has to withstand the effects of endogenous and extraneous contingencies as well as natural growth in the system load. The last stresses the system and causes contingency effects to be more severe. The most recent major blackout in North America is an example of contingencies having an adverse effect due to excessive loading [57]. Loading is a major issue mainly related to low voltage and voltage collapse.

Typically, the study of this issue is done by considering loading as a parameter in bifurcation studies. A Hopf bifurcation point in state space can be considered a boundary of a secure operation. If that point is passed the system becomes dynamically unstable. A margin for stability is set if the system is operated far from an HB point. However, due to power system economics and markets, this margin may be small at times. Nowadays, power systems are highly loaded during peak hours, making the study of nonlinear bifurcations and dynamic stability extremely important. When an HB point is exceeded, new stationary equilibrium points, periodic orbits, and chaotic behavior may occur. There are many works in this area [58–60]; however, a classical model for synchronous machines is generally considered, i.e., a classical swing-equation model. When more realistic models are used, such as the two-axis model, new issues and results arise that can give more insights about system stability. For example, when a SG is modeled by a two-axis model, an IEEE Type-1 exciter and a linear speed controller, new equilibrium points and bifurcations occur depending on the adjustments of the exciter set-point. In Figure 4.1, a PV-curve at the load bus of a single machine system is shown [40]. Solid and dotted lines correspond to stable and unstable trajectories, respectively. Basically, when the exciter set-point is adjusted to keep a constant terminal voltage at the SG (|\(V_t\)| is constant), there is a bifurcation before the limit point (LP). The new trajectory bifurcates upward from the PV curve and is unstable. When the exciter set-point
is kept fixed ($V_{ref}$ is fixed), this phenomenon does not occur. In general, the latter is a more realistic case as it is how SGs operate in a real system. Note that assuming a constant terminal voltage is a procedure generally used to simplify the analysis. This is done by using a power flow algorithm to determine equilibrium points. Thus, in this thesis, a constant exciter set-point is considered in bifurcation studies. In addition to the SG model, many studies have been done on the effects of load modeling. Static loads (e.g., constant-power, -current and -impedance) and dynamic loads (e.g., induction machines of various model orders) have been addressed [61–64]. With respect to the static-load models, it has been demonstrated that the constant-power-load model causes the lowest dynamic loading margin [62] and has a considerable impact on the saddle-node bifurcation (SNB) point [63, 64]. These are basically the most discussed issues in system stability; however, the addition of new technology mainly related to wind power generation has to be considered.

A high wind power system penetration, motivated by the concern due to global warming and the depletion of fossil fuels, would create a relative reduction of fossil-fuel plants which are mainly based on synchronous generators.
This reduction may reduce the system inertia making the system prone to instabilities. However, recent studies have proved the opposite; wind power generation does contribute positively to improve system stability. As a matter of fact, a high wind power penetration (about 20 or 30%) has been shown to reduce oscillations specifically related to inter-area modes [65–68]. When large groups of generators that form clusters in separated regions are weakly interconnected, inter-area oscillations arise and, if not damped, they can induce instabilities to the whole system. In general, wind power generator states have no participation in the inter-area modes [65]. Several wind power controllers have been proposed to damp power system oscillations either by using signals proportional to frequency deviation [66] or by specialized designs [67, 68]. The use of power system stabilizers has also been proposed to improve the performance of wind farms by damping inter-area oscillations [69]. A recent study of the New Zealand power system, however, has shown that a high wind power penetration may have some negative impact on the system stability [70]. The New Zealand power system is characterized by two clusters of generators and loads located in the North and South Island. It is shown that a high wind power penetration degrades damping performance of the system, particularly following a contingency. This degradation is explained by the modification of generation dispatch and by network power flows changes.

In the next sections, the analysis of wind farms in power systems is presented. First, the multi-machine model used in this thesis is presented. Then, in order to identify the best places to locate wind farms, analytical sensitivity is presented with respect to any arbitrary parameter. Finally, bifurcations analysis and dynamic simulations are performed using several scenarios. Results obtained by the full formulation, the wind farm aggregated model and the wind farm reduced-order model are compared.

4.1 Multi-Machine Model

Consider a system with $n_{SG}$ synchronous generators, $n_{WTG}$ wind power generators and $n_b$ buses. Assume that every SG is modeled by a two-axis model and has an IEEE Type-1 exciter and a linear speed controller with droop. In addition, assume for simplicity that $X_d' = X_q'$. For the case of the WTGs,
consider a zero-axis model.

Define $b_{SG}$ as the set of buses at which the synchronous generators are connected. Thus, the $i^{th}$ generator is connected to the bus $b_{SG}(i)$ and its terminal voltage, voltage at bus $b_{SG}(i)$, is given by $\overline{V}_i = V_ie^{j\theta_i}$. Then, $\forall i = \{1, 2, ..., n_{SG}\}$

$$
T_{d0i} \frac{dE_{qi}'}{dt} = -E_{qi}' - (X_{di} - X_{di}')I_{di} + E_{fdi}
$$

$$
T_{q0i} \frac{dE_{di}'}{dt} = -E_{di}' + (X_{qi} - X_{qi}')I_{qi}
$$

$$
\frac{d\delta_i}{dt} = +w_i - w_s
$$

$$
\frac{2H_i d\omega_i}{\omega_s dt} = +T_{mi} - E_{di}'I_{di} - E_{qi}'I_{qi}
$$

$$
T_{Ei} \frac{dE_{fdi}}{dt} = -K_{Ei}E_{fdi} + V_{Ri}
$$

$$
T_{Fi} \frac{dR_{fi}}{dt} = -R_{fi} + \frac{K_{Fi}}{T_{fi}}E_{fdi}
$$

$$
T_{Ai} \frac{dV_{Ri}}{dt} = -V_{Ri} + \frac{R_{fi}}{K_{Ai}} + \frac{K_{Fi}}{T_{fi}}E_{fdi} + (V_{ref,i} - V_i)
$$

$$
T_{CHi} \frac{dT_{mi}}{dt} = -T_{mi} + P_{svi}
$$

$$
T_{SVi} \frac{dP_{svi}}{dt} = -P_{svi} + P_{Ci} - \frac{1}{R_{Di}} \frac{(w_i - w_s)}{w_s}
$$

The algebraic equations (4.10)-(4.11) can be alternatively represented by the equivalent circuit of Figure 4.2.

Define $b_{WTG}$ as the set of buses at which the wind power generators are connected. Thus, the $i^{th}$ generator is connected to the bus $b_{WTG}(i)$ and its terminal voltage, voltage at bus $b_{WTG}(i)$, is given by $\overline{V}_i = V_ie^{j\theta_i}$. Then, $\forall i = \{1, 2, ..., n_{WTG}\}$

$$
X_{SGi} = [E_{qi}', E_{di}', \delta_i, \omega_i, E_{fdi}, R_{fi}, V_{Ri}, T_{mi}, P_{svi}]^T
$$

$$
Y_{SGi} = [I_{qsi}, I_{dsi}]^T
$$

The algebraic equations (4.10)-(4.11) can be alternatively represented by the equivalent circuit of Figure 4.2.
∀i = \{1, 2, ..., n_{WTG}\}

\[
\begin{align*}
\frac{d\omega_{ri}}{dt} &= \frac{\omega_s}{2H_{Di}} [T_{mi} - X_{mi}I_{qsi}I_{dri} + X_{mi}I_{dsi}I_{qri}] \\
\frac{dx_{1i}}{dt} &= +K_{P1i} [P_{ref,i} - P_{gen,i}] \\
\frac{dx_{2i}}{dt} &= +K_{P2i} [P_{ref,i} - P_{gen,i}] + x_{1i} - I_{qri} \\
\frac{dx_{3i}}{dt} &= +K_{P3i} [Q_{ref,i} - Q_{gen,i}] \\
\frac{dx_{4i}}{dt} &= +K_{P4i} [Q_{ref,i} - Q_{gen,i}] + x_{3i} - I_{dri} \\
0 &= -V_{qri} + P_{gen,i} + V_{qsi} - (V_{qri}I_{qri} + V_{dri}I_{dri}) \\
0 &= -Q_{gen,i} + V_{dsi} \\
0 &= -V_{dsi} + X_{s}I_{qsi} - X_{s}I_{dsi} + X_{mi}I_{dri} \\
0 &= -R_{s}I_{dsi} + X_{s}I_{qsi} - X_{mi}I_{qri} \\
0 &= -V_{qri} + R_{r}I_{qri} - s_{i}X_{mi}I_{dsi} + s_{i}X_{ri}I_{dri} \\
0 &= -V_{dri} + R_{r}I_{dri} + s_{i}X_{mi}I_{qsi} - s_{i}X_{ri}I_{qri}
\end{align*}
\]

where \( s_{i} = (\omega_{s} - \omega_{ri}) / \omega_{s} \). The vectors of state and algebraic variables are

\[
\begin{align*}
X_{WTGi} &= [\omega_{ri}, x_{1i}, x_{2i}, x_{3i}, x_{4i}]^T \\
Y_{WTGi} &= [V_{qri}, V_{dri}, I_{qri}, I_{dri}, I_{qsi}, I_{dsi}, P_{gen,i}, Q_{gen,i}]^T
\end{align*}
\]

The algebraic equations (4.19)-(4.26) can alternatively be represented by the equivalent circuit of Figure 4.3.
In order to model the network constraints, define the admittance matrix of the system such that

$$Y_{bus}(i,i) = \sum_{\forall k} y_{shunt}^{k}(i,i) + \sum_{\forall j} y(i,j)$$

$$Y_{bus}(i,j) = -y(i,j) \quad (4.29)$$

where $y_{shunt}^{k}(i,i)$ is the admittance of the $k^{th}$ shunt element connected to bus $i$ and $y(i,j)$ is the equivalent admittance connected between buses $i$ and $j$. Obtain the conductance and susceptance matrices as $G_{bus} = \mathfrak{R}\{Y_{bus}\}$ and $B_{bus} = \mathfrak{I}\{Y_{bus}\}$, respectively. Then, $\forall i = \{1, 2, ..., n_b\}$

$$0 = -\mathfrak{R}\{\overline{I}_i\} + \sum_{\forall j} V_j (G_{bus}(i,j) \cos(\theta_j) - B_{bus}(i,j) \sin(\theta_j)) \quad (4.30)$$

$$0 = -\mathfrak{I}\{\overline{I}_i\} + \sum_{\forall j} V_j (B_{bus}(i,j) \cos(\theta_j) + G_{bus}(i,j) \sin(\theta_j)) \quad (4.31)$$

where $\overline{I}_i$ is the complex current injected to the bus $i$. This current may have three components depending on the type of connected element—synchronous generator, wind power generator or static load. Assuming a generic SG $k$ and a generic WTG $\ell$, where $k \in \{1, 2, ..., n_{SG}\}$ and $\ell \in \{1, 2, ..., n_{WTG}\}$, then

$$\mathfrak{R}\{\overline{I}_i\} = \begin{cases} I_{qk} \cos(\delta_k) + I_{dk} \sin(\delta_k) & \text{if } b_{SG}(k) = i \\ I_{q\ell} \cos(\theta_\ell) + I_{d\ell} \sin(\theta_\ell) - \left(\frac{V_q l_{q\ell} + V_d l_{d\ell}}{V_t}\right) \cos(\theta_\ell) & \text{if } b_{WTG}(\ell) = i \\ -P_{oi} V_i^{(pq-1)} \cos(\theta_i) - Q_{oi} V_i^{(qv-1)} \sin(\theta_i) & \text{if a load is connected} \end{cases}$$
\[ \text{Im}\{I_i\} = \begin{cases} 
I_{qk} \sin(\delta_k) - I_{dk} \cos(\delta_k) & \text{if } b_{SG}(k) = i \\
I_{qs\ell} \sin(\theta_{\ell}) - I_{ds\ell} \cos(\theta_{\ell}) - \left(\frac{V_{qr\ell} I_{qr\ell} + V_{dr\ell} I_{dr\ell}}{V_{\ell}}\right) \sin(\theta_{\ell}) & \text{if } b_{WTG}(\ell) = i \\
-P_{oi} V_i^{(q-1)} \sin(\theta_i) + Q_{oi} V_i^{(q-1)} \cos(\theta_i) & \text{if a load is connected} 
\end{cases} \]

Note that an exponential model has been considered for the load connected at bus \( i \) as \( P_{Li} + jQ_{Li} = P_{oi} V_i^{pe} + jQ_{oi} V_i^{qv} \).

### 4.2 Eigenvalues Sensitivity with Respect to an Arbitrary Parameter \( \kappa \)

Consider the system matrix \( A_{sys} \). Assume that its eigenvalues and corresponding right and left eigenvectors are defined by \( \mu_i, v_i \) and \( w_i \), respectively. Eigenvectors are defined as column vectors. Then, \( \forall i, j \)

\[ w_i^T A_{sys} = \mu_i w_i^T \quad (4.32) \]

\[ \Rightarrow w_i^T A_{sys} v_j = \mu_i w_i^T v_j \quad (4.33) \]

\[ A_{sys} v_j = \mu_j v_j \quad (4.34) \]

\[ \Rightarrow w_i^T A_{sys} v_j = \mu_j w_i^T v_j \quad (4.35) \]

Comparing Equations (4.33)-(4.35),

\[ (\mu_i - \mu_j) w_i^T v_j = 0 \quad (4.36) \]

\[ \Rightarrow \forall i \neq j, \ w_i^T v_j = 0 \quad (4.37) \]

Right and left eigenvectors are orthogonal. The calculation of the left eigenvectors can be normalized if \( w_i^T v_i = \delta_{ij} \), where \( \delta_{ij} \) is the Kronecker’s delta function. Letting \( V = [v_1, v_2, ..., v_n] \) be the matrix of right eigenvectors, then \( W = V^{-1} = [w_1, w_2, ..., w_n] \) is the matrix of left eigenvectors. Left eigenvectors calculated in this fashion are orthonormal to the set of right eigenvectors.

To obtain eigenvalues sensitivity with respect to an arbitrary parameter \( \kappa \)
κ, proceed as follows [71,72]:

\[ A_{sys}v_j = \mu_j v_j \]  
(4.38)

\[ \frac{\partial A_{sys}}{\partial \kappa} v_j + A_{sys} \frac{\partial v_j}{\partial \kappa} = \frac{\partial \mu_j}{\partial \kappa} v_j + \mu_j \frac{\partial v_j}{\partial \kappa} \]  
(4.39)

\[ w_j^T \frac{\partial A_{sys}}{\partial \kappa} v_j + w_j^T A_{sys} \frac{\partial v_j}{\partial \kappa} = w_j^T \frac{\partial \mu_j}{\partial \kappa} v_j + w_j^T \mu_j \frac{\partial v_j}{\partial \kappa} \]  
(4.40)

\[ w_j^T \frac{\partial A_{sys}}{\partial \kappa} v_j + (w_j^T A_{sys} - w_j^T \mu_j) \frac{\partial v_j}{\partial \kappa} = \frac{\partial \mu_j}{\partial \kappa} w_j^T v_j \]  
(4.41)

\[ \frac{\partial \mu_j}{\partial \kappa} = w_j^T \frac{\partial A_{sys}}{\partial \kappa} v_j \]  
(4.42)

When eigenvalues are complex, \( \mu_j = \mu_j^x + j\mu_j^y, \) \( v_j = v_j^x + jv_j^y \) and \( w_j = w_j^x + jw_j^y. \) Then, decomposing Equation (4.42) into its real and imaginary parts, the following two equations are obtained:

\[ \frac{\partial \mu_j^x}{\partial \kappa} = w_j^x \frac{\partial A_{sys}}{\partial \kappa} v_j^x - w_j^y \frac{\partial A_{sys}}{\partial \kappa} v_j^y \]  
(4.43)

\[ \frac{\partial \mu_j^y}{\partial \kappa} = w_j^x \frac{\partial A_{sys}}{\partial \kappa} v_j^y + w_j^y \frac{\partial A_{sys}}{\partial \kappa} v_j^x \]  
(4.44)

The system is made more stable if eigenvalues are moved to the left of the complex plane. Therefore, in this research, Equation (4.43) is of interest. \( A_{sys} \) is obtained numerically by eliminating algebraic variables. Thus, an explicit expression of its partial derivative with respect to the arbitrary parameter \( \kappa \) is not possible. However, this derivative can indirectly be obtained by using the Jacobian matrix and its sub-matrices \( A_s, B_s, C_s \) and \( D_s \) (see Appendix D).

\[ \frac{\partial A_{sys}}{\partial \kappa} = \frac{\partial}{\partial \kappa} \left( A_s - B_s D_s^{-1} C_s \right) \]  
(4.45)

\[ = \frac{\partial A_s}{\partial \kappa} - \frac{\partial B_s}{\partial \kappa} D_s^{-1} C_s - B_s \frac{\partial D_s^{-1}}{\partial \kappa} C_s - B_s D_s^{-1} \frac{\partial C_s}{\partial \kappa} \]  
(4.46)

where

\[ \frac{\partial D_s^{-1}}{\partial \kappa} = -D_s^{-1} \frac{\partial D_s}{\partial \kappa} D_s^{-1} \]  
(4.47)
4.3 New England Test System

Data of the New England test system is presented in Appendix G. For simplicity, the parameters of the exciter and governor at every SG are identical.

4.3.1 Small Signal Stability and Sensitivities

For the base case, there are four modes with a damping ratio less than 4.5% (see Appendix H for more details about damping ratio). These are considered critical modes in which just the angle $\delta$ and speed $\omega$ of the SGs participate. In Table 4.1, these modes, the damping ratio, the oscillation frequency and the SGs that participate in the modes through their angle and speed are presented. The most critical mode has a damping ratio of about 3%, which means that after a perturbation, the amplitude of the oscillations would be reduced to 57% in about 3 s and to 15% in about 10 s. Although this estimation is based on a linear model, this slow response can be visualized by time domain simulations when the voltage reference of the SG 3 is increased 1% (see Figure 4.4).

In order to study the importance of SG inertia in system dynamics, eigenvalues sensitivity with respect to the inertia of the $i^{th}$ SG is performed. Consider that the inertia of this generic SG is $H_i$. Eigenvalue sensitivities are defined by Equation (4.43) where

$$
\frac{dA_{sys}}{dH_i} = \frac{dA_s}{dH_i} - \frac{dB_s}{dH_i} D_s^{-1} C_s
$$

(4.48)

$A_s$ depends on $H_i$ through the equation of motion of the $i^{th}$ SG and its state variables $E'_{qi}$ and $E'_{di}$. $B_s$ depends on $H_i$ through the equation of motion of the $i^{th}$ SG and its algebraic variables $I_{qsi}$ and $I_{dsi}$. $C_s$ and $D_s$ do not

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$f_{osc}$ [Hz]</th>
<th>SG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1^i \pm j \mu_2^i = -0.3202 \pm j 7.9294$</td>
<td>4.04 %</td>
<td>1.2620</td>
<td>1, 3 and 8</td>
</tr>
<tr>
<td>$\mu_2^i \pm j \mu_2^2 = -0.2023 \pm j 6.6095$</td>
<td>3.06 %</td>
<td>1.0519</td>
<td>3, 5, 6 and 7</td>
</tr>
<tr>
<td>$\mu_3^i \pm j \mu_3^3 = -0.2216 \pm j 6.0620$</td>
<td>3.65 %</td>
<td>0.9648</td>
<td>5 and 9</td>
</tr>
<tr>
<td>$\mu_4^i \pm j \mu_4^4 = -0.1184 \pm j 3.4647$</td>
<td>3.42 %</td>
<td>0.5514</td>
<td>4, 5, 6, 7 and 9</td>
</tr>
</tbody>
</table>
Figure 4.4: State variables of the SG No.1 when the voltage reference of SG No.3 is increased 1%.

depend on $H_i$. Sensitivities are presented in Figure 4.5. Note that a positive sensitivity means that the corresponding eigenvalue is moved to the right of the complex plane when the inertia is increased. When the sensitivity is negative, then the corresponding eigenvalue is moved to the left when inertia is increased. The meaning of the sensitivity is reversed if the inertia is decreased. Note that the effect of increasing the inertia of SGs 5, 6, 7, 8 and 9 is beneficial for some modes and detrimental for others. For example, consider SG 5. An increase of inertia makes more stable the critical mode $\mu_3$, but it makes more unstable modes $\mu_1$, $\mu_2$ and $\mu_4$. Among all generators, it seems that a reduction of SG 3’s inertia can have, on average, a beneficial impact over critical modes. Here, critical modes $\mu_1$ and $\mu_2$ are moved to the left while modes $\mu_3$ and $\mu_4$ have a little displacement to the right. Note that this effect of inertia $H_3$ is not conventional because, in general, increasing inertia improves the system’s stability. This result is verified by calculating the equilibrium point and critical eigenvalues when the inertia of SG 3 is varied (see Table 4.2). Sensitivities are valid in a neighborhood around the base case parameters. The neighborhood’s size depends on the system and
Figure 4.5: Sensitivity of critical modes with respect to the SG’s inertia. 

parameters. Note that the neighborhood of $\frac{d\mu_4}{dH_3}$ is small and consequently not representative of the real system dynamics. As a result, mode $\mu_4$, instead of moving to the right, actually moves to the left.

As this thesis postulates that a negative load model suffices for representing a WTG in power systems, sensitivity with respect to the active power demand at every bus is performed. Consider that the active power demand at bus $i$ is $P_i$. Eigenvalue sensitivities are defined by Equation (4.43) where

$$\frac{dA_{sys}}{dP_i} = B_s D_s^{-1} \frac{dD_s}{dP_i} D_s^{-1} C_s$$  \hspace{1cm} (4.49)

Table 4.2: Sensitivity of critical eigenvalues with respect to inertia $H_3$

<table>
<thead>
<tr>
<th>$H_3$ [s]</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$\mu_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.8</td>
<td>$-0.320 \pm j7.93$</td>
<td>$-0.202 \pm j6.61$</td>
<td>$-0.222 \pm j6.06$</td>
<td>$-0.118 \pm j3.47$</td>
</tr>
<tr>
<td>32.5</td>
<td>$-0.350 \pm j7.97$</td>
<td>$-0.217 \pm j6.64$</td>
<td>$-0.217 \pm j6.07$</td>
<td>$-0.119 \pm j3.48$</td>
</tr>
<tr>
<td>30</td>
<td>$-0.380 \pm j8.62$</td>
<td>$-0.225 \pm j6.65$</td>
<td>$-0.215 \pm j6.07$</td>
<td>$-0.120 \pm j3.49$</td>
</tr>
<tr>
<td>27.5</td>
<td>$-0.357 \pm j8.91$</td>
<td>$-0.231 \pm j6.66$</td>
<td>$-0.212 \pm j6.08$</td>
<td>$-0.120 \pm j3.50$</td>
</tr>
</tbody>
</table>
$D_s$ depends on $P_i$, through the bus voltage at bus $i$, i.e., algebraic variables $V_i$ (voltage magnitude) and $\theta_i$ (voltage angle). $A_s$, $B_s$, and $C_s$ do not depend on $P_i$. Sensitivities for two PQ buses and two PV buses are presented in Table 4.3. Most of the buses show a negative sensitivity with respect to the active power demand such as bus 14 which is considered in the next section. However, their negative sensitivities are not as relevant as those found at buses 19, 20, 34 and 38 (which have the highest negative sensitivities). The negative sensitivities found at buses 19 and 20 mean that critical modes are moved to the left while the demand is increased. As WTGs inject power, negative demand, these PQ buses are not attractive for installing WTGs because critical eigenvalues may be moved to the right of the complex plane. Also, increasing the demand at buses 34 and 38 is beneficial but in this case the interpretation is different. These are PV buses. An increase of the active power demand can be alternatively expressed as a reduction of the power generated by the SG. An important issue must be stated at this time. Although a WTG does not have a major direct impact over the power system, a WTG indirectly impacts the power system by displacing generation in SGs and by changing network power flows [70,73]. Thus, the injection of power by WTGs should be performed together with a reduction of the power generated at bus 34 as this action is beneficial for the system stability (this fact must be considered in the power generation scheduling). Bus 38 is rejected as the reduction of the generated power from the SG can also move some critical modes to the right.

Table 4.3: Sensitivity of critical eigenvalues with respect to the active power demanded at bus $i$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\frac{d\mu_1^i}{dP_i}$</th>
<th>$\frac{d\mu_2^i}{dP_i}$</th>
<th>$\frac{d\mu_3^i}{dP_i}$</th>
<th>$\frac{d\mu_4^i}{dP_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>0</td>
<td>-0.0001</td>
<td>-0.0019</td>
<td>-0.0142</td>
</tr>
<tr>
<td>20</td>
<td>-0.0001</td>
<td>-0.0017</td>
<td>-0.0026</td>
<td>-0.0160</td>
</tr>
<tr>
<td>34</td>
<td>-0.0001</td>
<td>-0.0061</td>
<td>-0.0041</td>
<td>-0.0188</td>
</tr>
<tr>
<td>38</td>
<td>0.0009</td>
<td>0.0012</td>
<td>-0.0244</td>
<td>-0.0140</td>
</tr>
</tbody>
</table>
4.3.2 Low Wind Power Penetration

The impact of a low wind power penetration is studied by using a single equivalent WTG that produces from 0 to 5% (305.8 MW) of the total generated power. The equivalent WTG is arbitrarily connected at bus 14. Damping ratio and oscillation frequency of the critical modes are calculated for the mentioned wind power penetration (Figure 4.6). The movement direction of the modes when the generated power is increased is indicated by arrows. Note that, for this test system and its particular operating point, the effect of increasing the wind power penetration is on average detrimental. While the damping ratio of mode \( \mu_2 \) is increased, damping ratios of modes \( \mu_1, \mu_3 \) and \( \mu_4 \) are reduced and, thus, the attenuation of oscillations before a perturbation is worsened. With respect to the SGs’ inertia, the eigenvalues sensitivity with respect to the inertia of SG 3 and 5 is calculated for these increasing values of wind power penetration (see Figures 4.7 and 4.8). Observe that, on average, the impact of the SGs inertia is reduced when the wind power penetration is increased. For example, consider the critical modes at which \( H_3 \) and \( H_5 \) are most influential. Observe that \( \frac{d\mu_1^x}{dH_3} \) and \( \frac{d\mu_2^x}{dH_5} \), when the wind

![Figure 4.6: Critical modes when a WTG connected at bus 14 generates from 0% to 5% of the total generated power.](image-url)
Figure 4.7: Eigenvalue sensitivity with respect to $H_3$ when a WTG connected at bus 14 generates from 0% to 5% of the total generated power.

Figure 4.8: Eigenvalue sensitivity with respect to $H_5$ when a WTG connected at bus 14 generates from 0% to 5% of the total generated power.
power penetration is increased from 0 to 5%, are reduced from 0.0145 to 0.0138 and from 0.0184 to 0.0182, respectively.

When the wind power generation is performed together with a reduction of the generated power at bus 34, the dynamic behavior is improved. To show this effect, the generated power from the WTG is increased from 0 to 100 MW in intervals of 10 MW. Two cases are presented in Figure 4.9; in the first case, the WTG power is increased without executing any other action (black curve), and in the second case, the same power generated by the WTG is subtracted from the generation of the SG 5 which is connected at bus 34 (red curve). Note that in the second case all critical modes are improved having a higher damping ratio.

Note that these conclusions are valid in a neighborhood around the equilibrium point. When the power demand at bus 4 is varied, the impact of the equivalent WTG is different. Thus, define the power demand at bus 4 as a bifurcation parameter which is increased from its base value. The system’s HB point occurs at a loading parameter of 1715 MW, 1755 MW, 1790 MW, 1825 MW and 1850 MW when the wind power penetration is increased from 0 to 5%.

Figure 4.9: Critical modes when a WTG connected at bus 14 generates from 0 to 100 MW (black curve) and when the WTG’s power is compensated with a reduction of power of SG 5 (red curve).
1% to 5% in intervals of 1%, respectively.

Observe that the effect of increasing the wind power penetration is beneficial as the loading at the HB point is increased. The system is more robust as it is able to support higher loading. To sum up, although the impact of wind power generation over the system dynamics is negative around the initial equilibrium point (base case), its impact is beneficial as it increases the loading capability of the system. Moreover, displacing generation from SGs can have a significant impact on the system dynamics making critical modes more stable.

4.3.3 Aggregated and Reduced-Order Models

Three wind power scenarios are considered. In the first scenario a wind farm of 20 WTGs is connected at bus 14 (Wind Farm A). In the second scenario a wind farm of 50 WTGs is connected at bus 14 (Wind Farm B). In the third scenario two wind farms of 50 WTGs are connected at buses 14 and 19, respectively (Two Wind Farms B). Wind farms data is shown in Appendix I. Wind speed direction in reference to the turbines location is also shown in this appendix. In Wind Farm A, the wind speeds at the WTGs located at the first, second, third, fourth and fifth rows are 12, 11.6, 11.3, 10.9 and 10.6 m/s, respectively. In Wind Farm B, the wind speed at the WTGs located from the first to the tenth rows are the ones shown in Table 3.1. The WTGs located at the first row receive an incoming wind speed of 12 m/s. Four wind power models are taken into account (M1, M2, M3 and M4). M1 considers that every WTG is represented by a zero-axis model. M2 considers that every wind farm is represented by an aggregated model. M4 considers that every wind farm aggregated model is represented by a NL model.

Bifurcation analysis is performed and, for simplicity, five power transfers are defined as bifurcation parameters. All transfers consider the slack bus (bus 39) as the sending node, the place where the transfer is injected to the system. The receiving nodes, where the transfer is absorbed from the system, are buses 4, 8, 16, 20 and 24. One transfer at a time is considered. In Table 4.4, the loadings at the HB and SNB point for the three wind power scenarios, four wind farm models, and five power transfers are presented. There is a
Table 4.4: Loading in MW at bifurcation points for the three wind farm scenarios

<table>
<thead>
<tr>
<th>Transfer</th>
<th>Model</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>HB</td>
<td>SNB</td>
<td>HB</td>
</tr>
<tr>
<td>to bus 4</td>
<td>M1</td>
<td>1720</td>
<td>1980</td>
<td>1765</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>1725</td>
<td>1985</td>
<td>1770</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>1725</td>
<td>1985</td>
<td>1780</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>1730</td>
<td>1985</td>
<td>1790</td>
</tr>
<tr>
<td>to bus 8</td>
<td>M1</td>
<td>1620</td>
<td>1775</td>
<td>1645</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>1620</td>
<td>1775</td>
<td>1645</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>1620</td>
<td>1780</td>
<td>1655</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>1620</td>
<td>1780</td>
<td>1655</td>
</tr>
<tr>
<td>to bus 16</td>
<td>M1</td>
<td>1760</td>
<td>2110</td>
<td>1810</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>1765</td>
<td>2110</td>
<td>1820</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>1760</td>
<td>2110</td>
<td>1825</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>1770</td>
<td>2110</td>
<td>1835</td>
</tr>
<tr>
<td>to bus 20</td>
<td>M1</td>
<td>1845</td>
<td>2300</td>
<td>1870</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>1850</td>
<td>2300</td>
<td>1875</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>1845</td>
<td>2300</td>
<td>1875</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>1850</td>
<td>2300</td>
<td>1885</td>
</tr>
<tr>
<td>to bus 24</td>
<td>M1</td>
<td>1780</td>
<td>2120</td>
<td>1830</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>1785</td>
<td>2120</td>
<td>1840</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>1780</td>
<td>2120</td>
<td>1845</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>1790</td>
<td>2125</td>
<td>1860</td>
</tr>
</tbody>
</table>

M1: Every WTG represented by a two-axis model  
M2: Every WTG represented by a NL  
M3: Wind farm represented by an aggregated model  
M4: Wind farm aggregated model represented by a NL

great agreement between the four wind farm models. A slight increase of the loadings is observed when the model is aggregated and when a NL model is used. The largest discrepancy occurs between M1 and M4 when wind power scenario 3 and a power transfer to bus 24 are considered. The loading at the HB point with M1 and M4 is 1970 and 2025 MW, respectively. This difference corresponds to about 2.7% of error, which can be considered acceptable. Most importantly, the computational burden is dramatically reduced when
M4 is used. While 500 differential equations and 800 algebraic equations are employed with model M1, 4 algebraic equations are used with model M4. Note that when model M1 and M4 were used, the bifurcation analysis was performed in 425 and 1 minutes, respectively. A personal computer with a 2.2 GHz dual processor system and Matlab 6.5 were used. In this analysis, the most demanding task is to calculate the inverse of the Jacobian matrix. The simulation time increases exponentially with the number of variables. The eigenvalues’ pathways using the four models are very similar. In Figures 4.10 and 4.11, the eigenvalues’ pathways using the wind power scenario 3 and power transfer to buses 16 and 20 are shown. To sum up, the aggregation of WTGs in a wind farm and then its model order reduction does not impact the system dynamics. The reduced-order model mimics very well the the impact of WTGs in power system stability while obtaining an important reduction in dimensionality.

Time domain simulations are performed and, at first, wind power scenario 1 and models M1 and M4 are considered. Similarly to the assumptions made in Chapter 3, a wind gust is applied at \( t = 20 \) s which travels

![Eigenvalues pathway](image)

Figure 4.10: Dominant eigenvalues using wind power scenario 3 and a power transfer to bus 20.
along the WTGs. Thus, the new wind speed at every WTG is given by 
\( v_{\text{wind}}^{\text{new}} = v_{\text{wind}} + v_{\text{gust}} \). After the wind gust passes through a turbine’s blades, 
the gust’s wind speed is reduced by 2\%. Also, a traveling time of 2 s be-
tween the wind farm rows is considered. The wind gust causes wind speed 
variations of about ±1.5 m/s around the initial speed. The resulting equiva-

tent wind speed is shown in Figure 4.12. The equivalent wind speed at 
t = 0 s becomes 
\( v_{\text{wind}}(t=0) = 11.3018 \text{ m/s}. \)
The resulting first-order model 
for Wind Farm A, 
\[ P_{\text{gen}}^e = \beta_1 P_{\text{gen}}^e + \beta_2 v_{\text{wind}}^e + \beta_3, \]
has the following parameters:

\[
\begin{align*}
\beta_1 &= -0.2503 & \beta_2 &= 0.0536 & \beta_3 &= -0.4026 \\
\Rightarrow P_{\text{gen}}^e(t=0) &= -\frac{\beta_2 v_{\text{wind}}^e(t=0) + \beta_3}{\beta_1} = 0.8117
\end{align*}
\]

Using models M1 and M4, a comparison of a few state variables of SG 3 and the total power generated by the wind farm are presented. In Figure 4.13, variables related to active power are presented. Observe that there is a noticeable agreement between models M1 and M4. In Figure 4.14, variables related to voltage are presented. Observe the discrepancy between the models. Note that this issue is related with voltage and reactive power which,
in the proposed model, was treated in a simplified way. The reactive power output of every WTG was assumed equal to zero and, based on a few references [9, 11, 49, 54], the wind farm network losses were simply neglected. This issue can be clarified by observing bus voltages at buses 1, 8 and 14 of Wind Farm A and bus 14 of the New England System (Figure 4.15). Observe that the wind farm’s buses 8 and 14 increase their voltages when the wind speed increases. However, wind farm’s bus 1 and New England System’s bus 14 reduce their voltages. Moreover, observe the power sent from the wind farm to the PCC (Figure 4.16). While the estimation of active power is fair ($P_{\text{gen}}^e$), the reactive power absorbed by the wind farm is not estimated at all ($Q_{\text{gen}}^e = 0$ p.u.). Using the Ward Injection Method [74], the wind farm equivalent circuit of Figure 4.17 is found. The equivalent parameters and variables are calculated for the initial condition and when the total wind farm active power is maximum (see Table 4.5). Maximum power occurs at a simulation time of $t = 51.94$ s. Note that

a. Equivalent impedance and admittance are the same independent of the power generated by the WTGs.

Figure 4.12: Equivalent wind speed of wind power scenario 1.

98
Figure 4.13: Comparison of $\omega$, $T_m$ and $\delta$ of SG 3 and $P_{gen}^e$ using model M1 (black line) and M4 (red line).

Figure 4.14: Comparison of $E_{q}'$, $E_{d}'$, $E_{fd}$ and $R_f$ of SG 3 using model M1 (black line) and M4 (red line).
Figure 4.15: Bus voltages profile of buses 1, 8 and 14 of Wind Farm A and bus 14 of New England System (Model M1, wind power scenario 1).

Figure 4.16: Active and reactive powers using models M1 (black line) and M4 (red line). While $P_{\text{gen}}^e$ is considered with model M4, power sent from bus 1 of the Wind Farm A to the bus 14 of the NE System is considered with model M1.
Table 4.5: Voltage and power of wind farm equivalent circuit at initial condition and when wind farm active power is maximum

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>Maximum Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a = 0.9424 e^{-j0.0864}$ p.u.</td>
<td>$V_a = 0.9416 e^{-j0.0771}$ p.u.</td>
</tr>
<tr>
<td>$V_b = 0.9420 e^{-j0.1218}$ p.u.</td>
<td>$V_b = 0.9420 e^{-j0.1222}$ p.u.</td>
</tr>
<tr>
<td>$\bar{S} = 0.7846 - j0.0343$ p.u.</td>
<td>$\bar{S} = 0.9970 - j0.0559$ p.u.</td>
</tr>
<tr>
<td>$\sum_{k=1}^{20} P_{gen}^k = 0.8243$ p.u.</td>
<td>$\sum_{k=1}^{20} P_{gen}^k = 1.0614$ p.u.</td>
</tr>
</tbody>
</table>

b. $z$ and $y$ are equal to the parameters of the line that connects bus 1 of Wind Farm A and bus 14 of the New England System (PCC). $y_s$ depends on the wind farm network’s parameters and is obtained by the Ward injection method. Thus, $z = 0.002 + j0.04$ p.u., $y = j0.01$ p.u. and $y_s = j0.0161$ p.u.

c. $\bar{S}$ takes into account the total power generated by the WTGs and also the wind farm’s network losses.

d. The total reactive power absorbed by the wind farm is determined by the imaginary part of $\bar{S}$ and the admittance $y_s$.

In conclusion, when the WTGs increase their output, there is an increase of the reactive power absorption of the wind farm. While WTG’s terminal voltage increases with the increase of active power, wind farm terminal voltages decrease due to the increase of the wind farm’s reactive power absorption. Therefore, both active and reactive power losses of the wind farm should be included in the model. It seems that active power losses are not critical for the validity of the reduced-order model (observe Figures 4.13 and 4.16). On the other hand, reactive power losses have an important influence over the system. Although being low, reactive power absorbed by the wind farm can
create a different voltage profile if it is wrongly estimated. This has a direct impact on the variables associated with voltage, e.g., \( E_{fd}, E'_q \), and others.

In order to estimate the reactive power absorbed by the wind farm, selective modal analysis can be also used. Basically, relevant modes and variables related to the reactive power absorption of the wind farm have to be identified. This will be considered in future research. For now, the least squares method is used to find a curve that relates the reactive power absorption of the wind farm with the equivalent wind speed. The following model is proposed:

\[
\dot{Q}_{gen} = aQ_{gen}^e + bv_{wind}^e + c
\]  

where \( Q_{gen}^e \) is the total power injected by the wind farm to the grid. In other words, this power is the reactive power sent from bus 1 of the Wind Farm A to bus 14 of the New England System. The effect of the shunt admittance \( y_s \) is already taken into account in this power. To determine the model’s parameters, take \( m \) measurements from a time domain simulation using model M1. Create the following matrix equation in which every single row corresponds to the measurements at one particular time:

\[
\begin{bmatrix}
1\dot{Q}_{gen}^e \\
2\dot{Q}_{gen}^e \\
\vdots \\
m\dot{Q}_{gen}^e \\
y \in \mathbb{R}^{m \times 1}
\end{bmatrix}
= \begin{bmatrix}
1Q_{gen}^e \\
2Q_{gen}^e \\
\vdots \\
mQ_{gen}^e \\
y \in \mathbb{R}^{m \times 1}
\end{bmatrix}
= \begin{bmatrix}
1v_{wind}^e \\
2v_{wind}^e \\
\vdots \\
v_{wind}^e \\
M \in \mathbb{R}^{m \times 3}
\end{bmatrix}
= \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]  

\[
(4.51)
\]

where \( p_Q \) is the vector of parameters, \( y \) and \( M \) are the vector and matrix of measurements. Assume that there are more measurements than the number of parameters and that \( M \) is full column rank. Thus, \( M^TM \) is invertible. Note that \( \dot{Q}_{gen}^e \) is obtained by numerical differentiation. Thus, the parameters are calculated as

\[
p_Q = (M^TM)^{-1}M^Ty = \begin{bmatrix}
-0.2493 & -0.0045 & +0.0456
\end{bmatrix}^T
\]  

\[
(4.52)
\]

Time domain simulations are performed again with model M4 but including the modeling of the reactive power absorption by the wind farm. Figure 4.18 shows variables \( E'_q, E'_d, E_{fd} \) and \( R_f \) of SG 3. Observe that now there is
a good agreement between models M1 (black line) and M4 (red line). This model has been tested with other wind speed sequences obtaining satisfactory results. In the case of the wind power scenario 3, the parameters of the wind farms’ reduced-order models are obtained and are listed in Table 4.6.

Table 4.6: Parameters of the reduced-order models (wind scenario 3)

<table>
<thead>
<tr>
<th></th>
<th>Wind Farm A</th>
<th>Wind Farm B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>-0.2471</td>
<td>-0.2466</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>+0.1276</td>
<td>+0.1275</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.9340</td>
<td>-0.9335</td>
</tr>
<tr>
<td>$a$</td>
<td>-0.3088</td>
<td>-0.3401</td>
</tr>
<tr>
<td>$b$</td>
<td>-0.0071</td>
<td>-0.0155</td>
</tr>
<tr>
<td>$c$</td>
<td>+0.0716</td>
<td>+0.1534</td>
</tr>
<tr>
<td>$v_{\text{wind},0}$</td>
<td>11.0080</td>
<td>11.0080</td>
</tr>
<tr>
<td>$P_{\text{gen},0}^e$</td>
<td>+1.9046</td>
<td>+1.9060</td>
</tr>
<tr>
<td>$Q_{\text{gen},0}^e$</td>
<td>-0.0212</td>
<td>-0.0506</td>
</tr>
</tbody>
</table>

Figure 4.18: $E'_q$, $E'_d$, $E_{fd}$ and $R_f$ of SG 3 using model M1 (black line) and M4 (red line) including the reactive power absorption of the wind farm.
The results show that the reduced-order model’s variables follow the tendency of those of the full-order model (see Figures 4.19 and 4.20). There is an evident offset in the power sent to the grid. Using model M1, $P_{\text{to\ grid}}$ corresponds to the power transmitted from bus 1 of the Wind Farm B to bus 19 of the New England System (black line). Using model M4, $P_{\text{to\ grid}}$ corresponds to $P_{\text{gen}}$ of Wind Farm B (red line). This difference is due to the wind farm’s active power losses. As in reactive power losses, selective modal analysis will be considered in future research to improve the model by estimating the losses.

Note that each wind farm is represented by 250 differential equations and 400 algebraic equations when model M1 is employed. In contrast, 2 differential equations and zero algebraic equations are employed when model M4 is used. Nevertheless, note that the analyzed cases have a low wind power penetration. Wind power scenario 3 has a wind penetration less than 4%. In future research, higher wind power penetration will be considered. A WTG’s speed controller with frequency response will also be taken into account.

Figure 4.19: Wind power scenario 3, comparison of SG 3’s variables and reactive power injected to the grid. Results using M1 model (black line) and M4 model (red line).
4.3.4 Response Under Major Perturbations

The results so far indicate that no matter the system state, the active power generated by a wind farm depends basically on the wind farm’s average wind speed. Bifurcation analysis supports the idea that a simplified model based on a negative load model has no major impact on the system stability. Time domain simulations support the idea that system equilibrium point, the vector of state and algebraic variables, is not strongly affected by the simplified model. However, the system has been subjected only to wind speed variations. In this section, the behavior of the reduced-order model is tested when the system is subjected to major perturbations causing a notable change in the system equilibrium point.

A generator outage is a major disturbance that can cause low network voltages by disconnecting an important amount of reactive power. Also, the outage withdraws active power injection creating a re-distribution of power flows in the network. Generators G3, G4 and G6 are one of the most loaded and, thus, their sudden outages will be taken into consideration. Although not being as severe as a generator outage, line outage is also considered. The electrical line connecting buses 2 and 3 of the New England System has an active power flow of about 500 MW and a notable change in the network...
power flows can be created if the line goes out of service. The outage is applied at \( t = 30 \) s to one element at a time.

In Figure 4.21, the dynamic evolution of a few variables when SG 3 goes out of service is presented. Wind power scenario 1 is considered. The variables obtained using model M1 and model M4 are presented with black and red lines, respectively. Although the outage causes a voltage drop of about 4\% at bus 1 of the Wind Farm A, models M1 and M4 give similar results. There exists a small discrepancy when voltages \( V_1 \) and \( E_{fd} \) are compared. The reactive power absorption was modeled as a constant power and naturally the voltage reduction affects the wind farm losses. Although there is not an important discrepancy, voltage incidence over losses should be taken into account to improve the model. Similarly, the total active power injected by the wind farm is affected by this perturbation, but its impact over angular speed and torque is negligible. It seems that the oscillations of \( P_{to\, grid} \) do not have a major impact over the system’s response. The rest of the variables were also compared and good agreement between results using model M1 and M4 is found. The outage of SG 3 is also applied to wind power scenario 3 (see Figure 4.22) and similar conclusions are obtained. Note that the hypothesis that there is no major interaction between WTGs and SGs is verified in these cases. The major interaction between WTGs and the power system is the interchange of power. Observe that during the outage of SG 3, the most severe disturbance, the power injected by each wind farm is perturbed. However, their injected power quickly comes back to their unperturbed path which is defined by the wind speed profile. This unperturbed path is captured by the reduced-order model. Similar results are obtained when line 2-3, SG 4 and SG 6 go out of service.

The dynamic simulations are performed using Matlab’s solver \texttt{ode15s} which is a variable order solver based on numerical differentiation formulas in terms of backward differences. Results obtained with \texttt{ode15s} were successfully tested using Matlab’s solver \texttt{ode23s} which is based on a modified Rosenbrock formula of order 2 [75]. These are the only two solvers for DAEs system available in Matlab. Note that the New England System has several eigenvalues close to the imaginary axis which might affect the effectiveness of the solvers. Thus, to verify the New England test system’s results, other solvers should be considered in future research.
Figure 4.21: Comparison using M1 model (black line) and M4 model (red line) when SG 3 goes out of service at $t = 30$ s. Wind power scenario 1.

Figure 4.22: Comparison using M1 model (black line) and M4 model (red line) when SG 3 goes out of service at $t = 30$ s. Wind power scenario 3.
In this thesis, the modeling of wind farms to perform power system analysis is studied. Type-C wind turbine generators are considered. This type of wind generator consists of a turbine, a gearbox to increase the rotational speed, and a doubly-fed induction generator. Two ac/dc converters and a dc-link are used to feed the rotor circuit of the DFIG. The rotor power is drawn from the grid. For this configuration and based on time scale decomposition, two dynamic models are presented. In the first model, while rotor flux linkage dynamics are fully modeled, stator flux linkage dynamics are assumed to be infinitely fast. This representation is called the two-axis model. In the second model, both rotor and stator flux linkage dynamics are assumed to be infinitely fast. This is called the zero-axis model. In both models, a rotor speed controller, a reactive power controller and a pitch angle controller are considered. The turbine aerodynamic is represented by a static model which relates the wind speed, the rotor speed and the mechanical power extracted from the wind. The gearbox is assumed to be stiff and, therefore, a single-mass model is assumed. This single-mass model basically considers the turbine, the gearbox and generator’s shaft as a whole and represents them by a unique inertia constant. With respect to the controllers, the speed controller is designed to extract maximum power from the wind for a given wind speed. The reactive power controller is designed to follow a reference which is, in general, a zero reactive power output. Both speed and reactive power controller use proportional and integral control with an internal and external control loop. The internal loop is fast and involves the DFIG’s rotor currents. The external loop is slow and involves the DFIG’s active and reactive powers. The pitch angle controller is designed to limit the maximum active power output of the WTG and it also considers a proportional and integral control. Note that while the two-axis model contains 10 differential and 8 algebraic equations, the zero-axis model contains 8 differential and 8
algebraic equations. In most of the analysis, the wind speed, and therefore the active power output, is assumed to be within its limits. Consequently, the pitch angle controller is not taken into account. The numbers of differential equations of the two-axis and zero-axis models are reduced to 7 and 5, respectively. When every single WTG in a wind farm is represented by these models, the dimensionality of the power system model increases notably. In some cases, this can be an unattainable computational burden and reduced-order models are required.

Two-axis and zero-axis models are highly nonlinear and, whether a time domain simulation or static analysis is performed, a proper algorithm to initialize the model variables is required. Thus, an algorithm to calculate initial conditions is presented. This algorithm requires a turbine’s P-curves and the steady-state model of the DFIG. The P-curves are a graphical representation of the turbine aerodynamics and relate the wind speed, rotor speed and power extracted from the wind. The steady-state model of the DFIG is obtained by assuming that the mechanical torque given by the turbine and the rotor voltages given by the controllers are known. An equivalent circuit similar to the steady-state equivalent circuit of an induction machine is obtained. The only differences are that in the DFIG model two power injections are applied: mechanical power coming from the turbine and electrical power injected to the rotor circuit.

Several studies are performed using dynamic and steady-state models. The main conclusions can be summarized as follows:

a. The speed controller maximizes the power extraction from the wind (mechanical power) and feeds back the WTG’s power output (electrical power). Thus, this controller leads to a quasi-optimal equilibrium point. The difference between mechanical and electrical power is the DFIG’s stator and rotor losses. Note that mechanical losses are not considered in this work. For all the analyzed cases, DFIG’s losses are below 3% of the generated power.

b. A strong coupling in the q-axis rotor voltage and active power as well as the d-axis rotor voltage and reactive power is observed. Actually, this knowledge is used in the design of the speed and reactive power controllers. While the speed controller uses an internal loop with q-axis rotor current, the reactive power controller uses an internal loop with d-axis
rotor current. There is a weak cross-coupling between q-axis rotor voltage and reactive power and between d-axis rotor voltage and active power. This cross-coupling means that the action of one controller will affect the action of the other one.

c. The power capability of a DFIG is similar to that of a SG. By controlling the d-axis rotor voltage the DFIG can inject reactive power to the grid. Depending on the wind availability, when a WTG is injecting an important amount of active power, the available reactive power that the WTG can inject is reduced. In the worse loading case, a WTG can be set to produce no reactive power. Note that Type-A and Type-B WTGs absorb reactive power from the grid, which degrades the system bus profile. Thus, even if a WTG produces no reactive power, it still benefits the power system operation.

d. In a WTG, reactive power control is preferred over voltage control. It has been shown that due to the injection of power from WTGs in multiple points through an internal network (wind farm network), the use of voltage control at every WTG can lead to abnormal operating points. These abnormal points are characterized by an excessive amount of reactive power absorbed by the WTGs. Moreover, from the grid side, this control strategy is not attractive as the reactive power absorbed by the wind farm at the PCC is significant. On the other hand, reactive power control does not overload WTGs and minimizes both the losses and the reactive power absorbed by the wind farm. However, it creates overvoltage in the wind farm network buses. Nevertheless, in all simulated cases, the overvoltage is less than 8% which is not a severe issue.

e. When a WTG is represented by its two-axis model, modal analysis reveals that the rotor flux linkage dynamics are still much faster than the dynamics of other state variables. This justifies the use of the zero-axis model in which both stator and rotor flux linkage dynamics are represented by algebraic equations (they are assumed to be infinitely fast). Moreover, WTG’s variables participate in those modes that lie close to the real axis of the complex plane. Thus, WTG’s variables do not exhibit major oscillatory behavior when the system is perturbed. This phenomenon is observed during time domain simulations. In addition, WTG’s variables,
most of the time, do not participate in unstable modes. In power systems, there is generally a complex pair of eigenvalues that crosses the imaginary axis when the load is increased. This complex pair of eigenvalues defines the system HB point and is related with variables of the SG’s voltage controller. This is observed with and without a WTG. Thus, a WTG does not have a major impact on the system stability and the most important interaction with the system is the interchange of power. Nevertheless, when the gains of the internal loop of the speed and reactive power controllers lie in a certain range, the WTG’s variables have participation in this complex pair of eigenvalues and impact the eigenvalues pathway when the load is increased.

f. Based on the results of the modal analysis, a negative load model is proposed to represent a WTG in power systems. The full-order model is compared when the WTG is represented by a negative load and a hypothetical synchronous generator (HSG). In the case of a HSG, a PI-controller in cascade with an IEEE Type-1 exciter is employed to control reactive power. Bifurcation analysis is performed using loading as a bifurcation parameter. The eigenvalues pathway is compared. The results show that while the HSG has more dominant eigenvalues with a lower frequency than the base case, the negative load resembles very well the full-order model, especially the most dominant modes. To verify this conclusion, sensitivity analysis is performed. In all cases, a negative load model mimics properly the behavior of a WTG in the power system except when the gains of the internal loop of the speed and reactive power controllers lie in a certain range. With gains in this range the frequency of the most dominant modes is considerably reduced. Nevertheless, no alteration is observed in the system critical loading which remains approximately equal to the base case (loading at the HB point).

These results indicate that a negative load model suffices to represent a WTG in power system analysis. Note that this fact is valid only for a Type-C WTG and the models described in this thesis. Other controller and turbine models should be considered to generalize these results. The importance of validating a negative load model is to obtain a reduction of simulation time and model complexity while still obtaining acceptable results. In the case of static analysis, a typical negative load model represented by two
algebraic equations is required. In the case of time domain simulations, a
differential model that captures the active power dynamics is required. This
model has to use wind speed as input and generated power as both state
variable and output. The reactive power output is so far assumed to be
zero (algebraic equation). SMA is employed to obtain this simplified model.
There is a unique relevant mode which is related with the rotor angular speed
of the WTG, and consequently, with the active power output. When SMA
is applied, at least one state variable is required for every relevant mode.

As mentioned before, for a certain range of gains of the internal loop of
the speed and reactive power controllers, a negative load model does not pro-
vide satisfactory results with respect to the system stability (refer to modal
analysis). Using SMA, more modes related to the rotor angular speed are
found. In this case, more state variables are required and a single differential
equation based on a negative load model cannot be directly obtained. When
the gains lie in this critical range, the WTG’s inertia has an important inci-
dence in the model’s structure. It has been observed that when the inertia
is relatively low, the number of relevant modes and required state variables
of the reduced order model increases substantially. Reduced order models of
second and third order have been obtained. When the inertia is relatively
high, and thus the number of relevant modes is reduced, a single differential
equation based on a negative load modal may still provide acceptable results
in terms of the WTG’s power output estimation. To obtain a single differ-
etial equation model, from all relevant modes choose the one in which the
rotor angular speed, and therefore the active power output, has the largest
participation. To sum up, when controller gains are critical and the inertia
is high, a negative load model can still provide a good estimation of system
variables even when the system dynamics might be significantly altered.

When a simplified representation of a wind farm is needed, aggregation
techniques should be used. In this thesis, an aggregated model is proposed.
Basically, a single equivalent WTG is obtained by aggregating the mechanical
power extracted from the turbines. In other words, an equivalent mechanical
power is obtained by summing over all WTGs’ mechanical power. As a result,
an equivalent speed is defined. In addition, equivalent parameters and vari-
ables are defined using the same concept of adding powers. The equivalent
WTG has the same characteristic as individual WTGs having also equivalent
controllers. In addition, the equivalent WTG is reduced by using SMA. If a
wind farm has 100 WTGs, 500 differential equations and 800 algebraic equations are required when every WTG is modeled by its zero-axis model. When the proposed aggregated model is used, only 5 differential equations and 8 algebraic equations are needed. If the aggregated model is reduced, then 1 single differential equation (active power) and a single algebraic equation (reactive power) are needed. Another way to obtain an equivalent model for the wind farm is to represent every WTG by a reduced-order model. In the best case, every WTG can be represented by a negative load model. In the case of 100 WTGs, then 100 differential equations (for active power injection) and 100 algebraic equations (for reactive power injection) are required. In general, the following main conclusions are obtained from the aggregation of wind farms.

a. The results obtained using the full-order model, the aggregated model and reduced-order model do not show any large discrepancy. It seems that the wind speed averaging cancels a few oscillation modes which are not observed in the aggregated and reduced-order models. Still, the results are satisfactory showing a good agreement with those of the full-order model. The results are consistent also with those obtained by the Slootweg’s aggregation method. Although Slootweg’s method gives a good model, it seems that its main disadvantages are that variables have an offset due to the linearization of the power reference characteristic. Moreover, it seems that the results obtained with this method are smoother than those obtained with the full-order, aggregated and reduced-order models.

b. The proposed aggregated and reduced-order models are applied to the New England Test System. Four modes have a damping ratio inferior to 4.5% which is considered critical. Analytical sensitivity with respect to the SG’s inertia and active power injection is calculated. Results show that WTGs do not directly impact the system stability. On the contrary, WTGs impact system operation indirectly by displacing generation from SGs and by re-distributing power flows through the network. When power is injected by the WTGs, critical modes are degraded, but when the injection is performed together with a change in the generated power from SGs, the damping ratio of critical modes is positively increased.

c. Bifurcation analysis is performed to the New England test system. This
study has to do with how well the aggregated and reduced-order models retain the stability properties of the full-order model. Three wind power scenarios are considered: a wind farm of 20 WTGs connected at bus 14, a wind farm of 50 WTGs connected at bus 14 and two wind farms of 50 WTGs connected at buses 14 and 19, respectively. Four power transfers are considered. All of them consider bus 39 as the sending node (node where the power is injected). Four models are evaluated. The first model (M1) represents every WTG by its zero-axis model. The second model (M2) represents every WTG by a negative load model. The third model (M3) uses an equivalent WTG to represent the wind farms. The fourth model (M4) uses a reduced-order model to represent the wind farms. When the eigenvalues pathway and loading at HB and SNB points are compared, a good agreement among the four models is found. A slight increase of the loadings at HB and SNB points is found when the WTGs are aggregated and then when the equivalent model is reduced. In the worst case, the model M4 estimates the HB and SNB points with an error of about 2.7%.

d. Time domain simulations are performed. This study has to do with how well the aggregated and reduced-order models estimate the system variables. Wind power scenarios 1 and 3 are considered. When the wind passes through a turbine’s blades, the wind speed is reduced by 2%. A traveling time between wind farm rows of 2 s is considered. An incoming wind gust that travels through the wind farms creates wind speed variations of about $\pm 1.5$ m/s. The system variables are compared using models M1 and M4 showing good agreement. Also, major perturbations such as generator outages and line outages are considered. In all cases, the original variables are properly estimated when model M4 is used. Note that the hypothesis that there is no major interaction between WTGs and SGs is verified in this case. The major interaction between these machines is the interchange of power. Observe that during the outage of SG 3, the most severe disturbance, the power injected by the wind farms is perturbed but it rapidly comes back to its unperturbed path defined by the wind speed profile. This unperturbed path is captured by the reduced-order model.

e. The wind farm’s reduced-order model fails to properly estimate the system’s variables when the wind farm’s reactive power absorption is not
considered. Actually, both active and reactive power losses should be considered in order to improve the performance of the reduced-order model. The losses strongly depend on the wind speed profile; therefore, SMA can be utilized to estimate them. In this thesis, a first-order differential equation in terms of the reactive power injected by the wind farm is used. The equivalent wind speed is used as input. The model parameters are calculated using the least-square method. This reactive power model considerably improves the variables’ estimation when model M4 is used.

All these results prove that a negative load model for representing WTGs in a power system is possible. Replacing either a two-axis or zero-axis model of a WTG with a negative load neither considerably alters the original system dynamics nor modifies the system variables. This is beneficial due to the reduction of simulation time and model complexity. In the largest case, it is shown that two wind farms that in total are represented by 500 differential equations and 800 algebraic equations can be represented by just 4 differential equations. There are other factors that should be included in this study such as equivalency during short-circuits, different control schemes, wind farm loss estimation, and wind farm central control, among others. All these issues are left for future research and are listed next.

5.1 Future Research

a. In the two-axis model, consider a complete model for the field oriented control and then reduce the model’s order. Establish a simplified model to dynamically estimate the shift angle between the WTG’s q-d axis reference and the system reference.

b. Other turbine models and control systems should be evaluated to test the validity of using a negative load model.

c. Include WTGs with frequency response. Basically, an input related to frequency deviation is included in the external loop of the speed controller. More interaction between WTGs and SGs is expected.

d. A central control that coordinates the active power and reactive power controllers at every WTG should be studied. Minimization of losses and
maximization of power extraction from the wind may be considered as global objectives.

e. Obtain a new reduced-order model that considers both active and reactive power losses of the wind farm. Use SMA for that purpose. Also, consider voltage incidence over losses and explore the utilization of voltage at PCC as an input in the reduced-order model. Moreover, perform short-circuit analysis to estimate equivalent parameters that capture the impact of the wind farm network on the short-circuit currents and on the system variables during time domain simulations.

f. Perform more studies on the influence of the controller gains over the system stability and their impact on the proposed aggregated and reduced-order models. Evaluate different tuning techniques to calculate PI-controller gains. Then, evaluate the system stability for the different set of parameters found.
APPENDIX A

ACTIVE AND REACTIVE POWER CONTROL

Assume that the $d$-axis is oriented along the stator flux axis, i.e., $\psi_s = \psi_{ds}$ with $\psi_{qs} = 0$. In addition, neglect $R_s$ and use Equations (2.10)–(2.11) in steady state to get $V_{ds} = \psi_{qs} = 0$ and $V_{qs} = \psi_{ds} = V_D$. Consider the flux equation (2.14) to obtain

$$I_{qs} = \frac{X_m}{X_s} I_{qr} \quad I_{ds} = \frac{X_m}{X_s} I_{dr} - \frac{V_D}{X_s}$$

(A.1)

Then, the complex power leaving the generator’s stator is

$$P_s + jQ_s = (V_{ds} I_{ds} + V_{qs} I_{qs}) + j(V_{qs} I_{ds} - V_{ds} I_{qs})$$

(A.2)

$$= \left(\frac{X_m}{X_s} V_D I_{qr}\right) + j \left(V_D \frac{X_m I_{dr} - V_D}{X_s}\right)$$

(A.3)

It turns out that the control of active and reactive power can be performed independently varying $I_{qr}$ and $I_{dr}$, respectively. The alignment of the $d$-axis and stator flux axis is obtained by using field-oriented control. In this research, a very fast field-oriented control is assumed and, therefore, an ideal shift-angle transformer which keeps a zero angle at the DFIG’s terminal is considered (see Figure 2.8). This angle is the shift angle between the variables in machine reference and network reference. In commercial software, such as PSS/E, this angle is calculated by a closed-loop feedback controller. However, it has been observed that the results are very sensitive to the controller’s gains. In future research, the dynamic of the field-oriented control will be considered and a simplified model to dynamically calculate this angle will be obtained.

The active power controller is designed to extract maximum power from the wind. When the pitch angle is constant, power extraction depends on both $v_{\text{wind}}$ (uncontrollable) and $\lambda$ (controllable) which is defined in terms of $\omega_r$. Therefore, controlling $\omega_r$ we can move along the power curve for
a given wind speed to maximize the power. Tracing a curve through the maximum power points for every given wind speed of Figure 2.1 (see also Figure 2.15), a one-to-one correspondence between optimal power and rotor speed is obtained. This correspondence and the minimum speed (cut-in), typically $0.7 \times \omega_{\text{rated}}$, and maximum speed, $1.2 \times \omega_{\text{rated}}$, due to converter ratings [17], is used to define a power reference (tracking curve). If the speed exceeds its maximum, pitch-angle control must be performed. Based on Equation (A.3), a PI controller with an internal $I_{qr}$-control loop is considered (Figure 2.4). In the reactive power controller, a PI controller with an internal $I_{dr}$-control loop is used (see Figure 2.5) [25].
The parameters of the 4-bus test system (Figure 2.23) are in per unit unless otherwise is specified. A common base has been chosen.

\textit{Doubly-fed induction generator}

\( X_m = 3.5092, \ X_s = 3.5547, \ X_r = 3.5859, \ \omega_s = 120\pi \ [rad/s], \ R_s = 0.01015, \)
\( R_r = 0.0088, \ H = 4 \ [s], \ p = 4, \ \rho = 1.225 \ [kg/m^3], \ R = 15 \ [m], \ S_b = 1 \ [MVA], \ C = 3.2397 \times 10^{-9} \ [s^3/rad^3], \ k = 1/45, \ K_{P_1} = K_{P_2} = K_{P_3} = K_{P_4} = 1, \ K_{I_1} = K_{I_2} = K_{I_3} = K_{I_4} = 5, \ Q_{ref} = 0 \)

\textit{Synchronous machine}

\( X_d = 2.2, \ X_q = 1.76, \ X'_d = 0.2, \ X'_q = 0.2, \ T_{d0} = 8 \ [s], \ T_{q0} = 1 \ [s], \ H = 10 \ [s], \)
\( K_E = 1, \ T_E = 0.7 \ [s], \ K_F = 0.03, \ T_F = 1 \ [s], \ K_A = 200, \ T_A = 0.04 \ [s], \)
\( T_G = 5 \ [s], \ V_{ref} = 1.0078 \)

\textit{Network}

Line 1: \( R_1 = 0.03, \ X_1 = 0.10 \)
Line 2: \( R_2 = 0.10, \ X_2 = 0.10 \)
Load: \( p_v = 0, \ q_v = 0, \ \frac{f_0}{\omega_0} = 5 \)
Transformer: \( X_T = 0.07 \)
APPENDIX C

DAE OF THE FOUR-BUS TEST SYSTEM

Consider the 4-bus system of Figure 2.23. Assume an exponential model for the load as \( P_L = P_0 V_L^{k_p} \) and \( Q_L = Q_0 V_L^{k_q} \). The synchronous generator (SG) is represented by a two-axis model \([29]\) and the transient reactance at both \( q\)- and \( d\)-axis is the same, i.e., \( X_d' = X_q' \). An IEEE Type-1 exciter and a linear speed governor without droop \([37]\) are considered. In addition, assume that the wind speed is such that the rotor speed is between the cut-in and maximum speed. Consequently, the rotor speed controller is operating over the optimal tracking curve and no pitch control is required. The optimal curve is defined as \( P_{ref} = C \omega_r^3 \) \([pu]\). Then, the complete set of differential algebraic equations for the wind power generator model and the four-bus system is the following.

**WTG Differential Equations**

\[
\frac{dE'_D}{dt} = - \frac{1}{T_0} \left( E'_D + (X_s - X'_s) I_{ds} \right) + \left( \omega_s \frac{X_m}{X_r} V_{dr} - (\omega_s - \omega_r) E'_D \right) \quad (C.1)
\]

\[
\frac{dE'_D}{dt} = - \frac{1}{T_0} \left( E'_D - (X_s - X'_s) I_{qs} \right) - \left( \omega_s \frac{X_m}{X_r} V_{qr} - (\omega_s - \omega_r) E'_D \right) \quad (C.2)
\]

\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{2H_D} \left[ T_m - E'_D I_{ds} - E'_D I_{qs} \right] \quad (C.3)
\]

\[
\frac{dx_1}{dt} = K_{I1} \left[ P_{ref} - P_{gen} \right] \quad (C.4)
\]

\[
\frac{dx_2}{dt} = K_{I2} \left[ K_{P1} \left( P_{ref} - P_{gen} \right) + x_1 - I_{qr} \right] \quad (C.5)
\]

\[
\frac{dx_3}{dt} = K_{I3} \left[ Q_{ref} - Q_{gen} \right] \quad (C.6)
\]

\[
\frac{dx_4}{dt} = K_{I4} \left[ K_{P3} \left( Q_{ref} - Q_{gen} \right) + x_3 - I_{dr} \right] \quad (C.7)
\]
SG Differential Equations

\[ T_{d0}^\prime \frac{dE_q'}{dt} = -E_q' - (X_d - X_d')I_d + E_{fd} \quad \text{(C.8)} \]

\[ T_{q0}^\prime \frac{dE_d'}{dt} = -E_d' + (X_q - X_q')I_q \quad \text{(C.9)} \]

\[ \frac{d\delta}{dt} = w - w_s \quad \text{(C.10)} \]

\[ \frac{2H}{\omega_s} \frac{d\omega}{dt} = T_m - E_d'I_d - E_q'I_q \quad \text{(C.11)} \]

IEEE Type-1 Exciter

\[ T_E \frac{dE_{fd}}{dt} = -K_E E_{fd} + V_R \quad \text{(C.12)} \]

\[ T_F \frac{dR_f}{dt} = -R_f + \frac{K_F}{T_F} E_{fd} \quad \text{(C.13)} \]

\[ \frac{T_A}{K_A} \frac{dV_R}{dt} = -\frac{V_R}{K_A} + R_f - \frac{K_F}{T_F} E_{fd} + (V_{ref} - V_s) \quad \text{(C.14)} \]

Linear Speed Governor without Droop

\[ T_G \frac{dP_m}{dt} = \omega_s - w \quad \text{(C.15)} \]

\[ T_m = P_m \frac{\omega_s}{w} \quad \text{(C.16)} \]

or

Linear Speed Governor with Droop

\[ \frac{dT_m}{dt} = \frac{1}{T_{CH}} (-T_m + P_{sv}) \quad \text{(C.17)} \]

\[ \frac{dP_{sv}}{dt} = \frac{1}{T_{SV}} \left(-P_{sv} + P_C - \frac{1}{R_D} \frac{w - w_s}{w_s}\right) \quad \text{(C.18)} \]
WTG Algebraic Equations

\[ 0 = -V_{qr} + K_{P2} (P_{ref} - P_{gen}) + x_1 - I_{qr} + x_2 \]  \hspace{1cm} (C.19)
\[ 0 = -V_{dr} + K_{P4} (Q_{ref} - Q_{gen}) + x_3 - I_{dr} + x_4 \]  \hspace{1cm} (C.20)
\[ 0 = -P_{gen} + E_{dD}^r I_{ds} + E_{qD}^r I_{qs} - R_s (I_{ds}^2 + I_{qs}^2) - (V_{qr} I_{qr} + V_{dr} I_{dr}) \]  \hspace{1cm} (C.21)
\[ 0 = -Q_{gen} + E_{qD}^r I_{ds} - E_{dD}^r I_{qs} - X_s (I_{ds}^2 + I_{qs}^2) \]  \hspace{1cm} (C.22)
\[ 0 = -I_{dr} + \frac{E_{qD}^r}{X_m} + \frac{X_m}{X_r} I_{ds} \]  \hspace{1cm} (C.23)
\[ 0 = -I_{qr} - \frac{E_{dD}^r}{X_m} + \frac{X_m}{X_r} I_{qs} \]  \hspace{1cm} (C.24)

Network, Load and Machines Algebraic Equations

\[ E_q' - j E_d' = j X_d' (I_q - j I_d) + V_S e^{j(\theta_S - \delta)} \]  \hspace{1cm} (C.25)
\[ E_q' - j E_d' = (R_1 + j [X_d' + X_1 + X_T]) (I_q - j I_d) + V_D e^{j(\theta_D - \delta)} \]  \hspace{1cm} (C.26)
\[ E_qD' - j E_dD' = (R_s + j X_s') (I_{qs} - j I_{ds}) + V_D e^{j\theta_D} \]  \hspace{1cm} (C.27)
\[ V_D e^{j\theta_D} = (R_2 + j X_2) I_L + V_L e^{j\theta_L} \]  \hspace{1cm} (C.28)
\[ P_L - j Q_L = P_0 V_L^{pv} - j Q_0 V_L^{qv} = V_L e^{-j\theta_L} I_L \]  \hspace{1cm} (C.29)

where

\[ I_L = (I_d + j I_q) e^{j(\delta - \frac{\pi}{2})} + I_{qs} - j I_{ds} - \frac{V_{qr} I_{qr} + V_{dr} I_{dr} e^{j\theta_D}}{V_D} \]

Equations (C.25)-(C.29) are complex and obtained from the equivalent circuit of Figure 2.23. For each one, obtain two equations by taking the real and imaginary part. The terms \( x_1, x_2, x_3 \) and \( x_4 \) are intermediate variables associated to the speed and reactive-power controllers of the WTG. Both a linear speed governor without and with droop are used for bifurcation analysis and dynamic simulations, respectively. For steady-state analysis, the angle \( \delta \) of the SG is calculated using the steady-state model of the generator [29]. Thus,

\[ \delta = \theta_S - \sin^{-1} \left( \frac{-X_q I_q}{V_S} \right) \]  \hspace{1cm} (C.30)
APPENDIX D

SMALL-SIGNAL STABILITY ANALYSIS

In general, a power system is modeled by a set of differential algebraic equations (DAEs) as

\[ \dot{x} = f(x, y, \alpha) \]  
\[ 0 = g(x, y, \alpha) \]

where \( x \in \mathbb{R}^{n_d \times 1} \) is the vector of differential variables, \( y \in \mathbb{R}^{n_a \times 1} \) is the vector of algebraic variables and \( \alpha \in \mathbb{R}^{n_p \times 1} \) is the vector of parameters. The terms \( n_d, n_a \) and \( n_p \) are the number of differential variables, algebraic variables and parameters, respectively, and \( g_y = \frac{\partial g(x, y, \alpha)}{\partial y} \) is nonsingular along the solution.

A linear approximation of the set of DAEs is required in order to perform a modal analysis. Use the generic form shown in Equations (D.1)-(D.2) and, given \( \alpha \), linearize the system equations around an equilibrium point \( (x_0, y_0) \).

\[ \Delta \dot{x} = A_s \Delta x + B_s \Delta y \]  
\[ 0 = C_s \Delta x + D_s \Delta y \]

Here, \( A_s, B_s, C_s \) and \( D_s \) depend on \( x_0, y_0 \) and \( \alpha \). Using Kron’s reduction, the algebraic equations are eliminated to obtain

\[ \Delta \dot{x} = (A_s - B_s D_s^{-1} C_s) \Delta x = A_{sys} \Delta x \]

Eigenvalues of \( A_{sys} \) determine the local stability of the operating point. Note that if all the eigenvalues are located on the left half of the complex plane, the equilibrium point is stable. If some eigenvalue has a positive real part, the equilibrium point is unstable. If a complex pair of eigenvalues crosses the imaginary axis to the right half while the others remain on the left half plane, then the crossing point is called a Hopf bifurcation (HB) point and the system becomes critically stable.
Nonlinearity and parameter dependence are inherent characteristics in power systems. These are autonomous and typically described by a set of differential algebraic equations. Branching or bifurcation diagrams can be used to understand system behavior. Basically they describe the trajectory of either stationary equilibrium points or periodic orbits in the state space when some parameters are varied, i.e., bifurcation parameters. The number of solutions and the system stability may change depending on the bifurcation parameter value and the branch that the system follows (Figure E.1). The local stability of the system around a stationary equilibrium point depends on the location of the system eigenvalues. Definitions such as stable node, unstable node, saddle, stable focus, unstable focus and center are considered [76]. The stability of system periodic orbits can be determined using Floquet multipliers [77]. When a single pair of complex eigenvalues crosses the imaginary axis to the positive real side and no other eigenvalue has a nonnegative real part, then “there is a birth of limit cycles” [77]. This bifurcation defines new trajectories of periodic orbits. The crossing point is called a Hopf bifurcation point.

Saddle-node bifurcation points (SNB) and turning points are important for representing system behavior in state space. An SNB point is characterized by a zero eigenvalue and receives its name from the collision of a saddle and a node. Turning points, also known as limit points (LP), reflect change in the direction of state variable trajectories. For example, in the classical PV curve, the limit point corresponds to the nose of the curve. Note that an LP does not always coincide with an SNB point [77]. Moreover, when the system hits an SNB point, new stationary equilibrium points or periodic orbits can arise [58].

Power system analysis methods, thus, must be carefully designed due to complex system behavior as well as to the appearance of new trajectories.
Figure E.1: Bifurcation diagram of an hypothetic state variable $x$ in terms of the bifurcation parameter $\lambda$.

in state space. Because all possible trajectories must be identified in order to avoid an unexpected system behavior, appropriate methods, models, and procedures to perform power system analysis are needed.
Consider a linear dynamic system of \( n + m \) states as

\[
\dot{x} = Ax
\]  

(F.1)

Assume that there are \( h \) modes of interest. By using participation factors, \( n \) states are found to be related to the modes of interest. Consequently, there are \( m \) states not related to the relevant modes. Note that \( h \leq n \), i.e., there is at least one state associated to each mode. Define, \( r \in \mathbb{R}^{n \times 1} \) as the vector of relevant states and \( z \in \mathbb{R}^{m \times 1} \) as the vector of less relevant states. Thus, the linear dynamic system model can be written as

\[
\begin{bmatrix}
\dot{r} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
r \\
z
\end{bmatrix}
\]  

(F.2)

This technique looks for a simplified representation of the less relevant dynamics. A subsystem of the less relevant dynamics can be stated as

\[
\begin{align*}
\dot{z} &= A_{22}z + A_{21}r \\
y &= A_{12}z
\end{align*}
\]  

(F.3)
(F.4)

where \( y \) is the subsystem’s output and, thus, the relevant dynamics are represented by \( \dot{r} = A_{11}r + y \) (see Figure F.1). The subsystem’s state variables and input are \( z \) and \( r \), respectively. Then, for \( t \geq t_0 \), the analytical solution for \( z \) is

\[
z(t) = e^{A_{22}(t-t_0)}z(t_0) + \int_{t_0}^{t} e^{A_{22}(t-\tau)}A_{21}r(\tau)d\tau
\]  

(F.5)

Considering the relevant modes, \( r(t) \) can be expressed as
Figure F.1: Selective modal analysis technique.

\[ r(t) = \sum_{i=1}^{h} \ell_i v_i e^{\lambda_i t} \]  \hspace{1cm} (F.6)

where \( \lambda_i \) is the \( i^{th} \) relevant eigenvalue and \( v_i \) is its corresponding eigenvector. Note that \( v_i \) just considers the components of the relevant variables. \( \ell_i \) is an arbitrary constant. Thus, replacing (F.6) in (F.5)

\[
z(t) = e^{A_{22}(t-t_0)} z(t_0) + \int_{t_0}^{t} e^{A_{22}(t-\tau)} A_{21} \sum_{i=1}^{h} \ell_i v_i e^{\lambda_i \tau} d\tau
\]  \hspace{1cm} (F.7)

\[
z(t) = e^{A_{22}(t-t_0)} z(t_0) + \sum_{i=1}^{h} \ell_i e^{A_{22}t} \int_{t_0}^{t} e^{(\lambda_i I - A_{22})\tau} d\tau A_{21} v_i
\]  \hspace{1cm} (F.8)

\[
z(t) = e^{A_{22}(t-t_0)} z(t_0) + \sum_{i=1}^{h} \ell_i e^{A_{22}t} (\lambda_i I - A_{22})^{-1} [e^{(\lambda_i I - A_{22})t} - e^{(\lambda_i I - A_{22})t_0}] A_{21} v_i
\]  \hspace{1cm} (F.9)

\[
z(t) = e^{A_{22}(t-t_0)} z(t_0) + \sum_{i=1}^{h} \ell_i (\lambda_i I - A_{22})^{-1} [e^{\lambda_i t} - e^{A_{22}(t-t_0)} e^{\lambda_i t}] A_{21} v_i
\]  \hspace{1cm} (F.10)

\[
z(t) = e^{A_{22}(t-t_0)} \left[ z(t_0) - \sum_{i=1}^{h} \ell_i (\lambda_i I - A_{22})^{-1} A_{21} v_i e^{\lambda_i t} \right] + \sum_{i=1}^{h} \ell_i (\lambda_i I - A_{22})^{-1} A_{21} v_i e^{\lambda_i t}
\]  \hspace{1cm} (F.11)
The analytical solution for the subsystem’s output is

\[ y(t) = A_{12} e^{A_{22}(t-t_0)} \left[ z(t_0) - \sum_{i=1}^{h} \ell_i (\lambda_i I - A_{22})^{-1} A_{21} v_i e^{\lambda_i t} \right] \]

\[ + \sum_{i=1}^{h} \ell_i A_{12} (\lambda_i I - A_{22})^{-1} A_{21} v_i e^{\lambda_i t} \]

\[ = A_{12} e^{A_{22}(t-t_0)} \left[ z(t_0) - \sum_{i=1}^{h} \ell_i (\lambda_i I - A_{22})^{-1} A_{21} v_i e^{\lambda_i t} \right] \]

\[ + \sum_{i=1}^{h} \ell_i H(\lambda_i) v_i e^{\lambda_i t} \]

(F.12)

where \( H(\lambda_i) = A_{12} (\lambda_i I - A_{22})^{-1} A_{21} \). In the frequency domain, \( H(s) \) is the transfer function of the subsystem which can be derived from Equation (F.2) by applying Laplace transformation.

\[ y(s) = H(s)r(s) \text{ where } H(s) = A_{12} (sI - A_{22})^{-1} A_{21} \]  

(F.14)

Note that if \( A_{22} \) is negative definite, in Equation (F.13), the natural response will decay and the subsystem’s output will correspond to the forced response. Moreover, assume \( \exists M_0 \) such that

\[ y(t) = \sum_{i=1}^{h} \ell_i H(\lambda_i) v_i e^{\lambda_i t} \triangleq M_0 \sum_{i=1}^{h} \ell_i v_i e^{\lambda_i t} = M_0 r(t) \]

(F.15)

\[ \Rightarrow \forall i = \{1, 2, ..., h\}, (M_0 - H(\lambda_i)) v_i = 0 \]

(F.16)

\[ \Rightarrow M_0 \begin{bmatrix} v_1, v_2, ..., v_h \end{bmatrix} = [H(\lambda_1)v_1, H(\lambda_2)v_2, ..., H(\lambda_h)v_h] \]

(F.17)

A sufficient condition for \( M_0 \) to exist is \( \text{rank}\{V_h\} = h \). The solution for \( M_0 \) is unique if and only if \( \text{rank}\{V_h\} = h = n \). If there are multiple solutions, i.e., \( h < n \), degrees of freedom can be eliminated by considering additional conditions to the problem. These conditions affect the convergence properties of numerical techniques used to solve the set of equations. One condition that has shown a good convergence is to consider a minimum norm for \( M_0 \) [44]. Finally, a reduced order model is obtained as \( \dot{r}(t) = (A_{11} + M_0) r(t) \) (see
Figure F.1). The main advantage of selective modal analysis is that the
eigenvalues associated to the relevant dynamics are the same in both the full
and reduced order model.

Example 1: A single real eigenvalue

Consider the following linear system and assume that the relevant mode
is the slowest one.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
-4 & 1 & 1 \\
0 & -2 & 1 \\
2 & -1 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]  
(F.18)

In matrix form, eigenvalues and right- and left-eigenvectors of \( A \) are

\[
\Lambda = \begin{bmatrix}
\lambda_a & 0 & 0 \\
0 & \lambda_b & 0 \\
0 & 0 & \lambda_c
\end{bmatrix}
= \begin{bmatrix}
-4.618 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & -2.382
\end{bmatrix}
\]  
(F.19)

\[
R = \begin{bmatrix}
0.6826 & -0.4472 & -0.3361 \\
0.2607 & -0.8944 & -0.8798 \\
-0.6826 & 0 & 0.3361
\end{bmatrix}
\]  
(F.20)

\[
L = R^{-1} = \begin{bmatrix}
1.3102 & -2.2361 & 2.6614 \\
-0.6551 & 0 & -1.3307 \\
-0.4049 & -2.2361 & 2.1532
\end{bmatrix}
\]  
(F.21)

Mode of interest is \( \lambda_1 = -2 \) (\( h = 1 \)). Participation factors are defined by

\[
p_{ij} = \frac{|r_{ij}| |l_{ij}|}{\sum_{i} |r_{ij}| |l_{ij}|}
\]  
(F.22)

Here, \( p_{ij} \) is a measure of the participation of state \( i \) in the dynamic mode \( j \). Thus,

\[
P = \begin{bmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{bmatrix}
= \begin{bmatrix}
0.6667 & 1 & 0.3207 \\
0.1273 & 0 & 0.4198 \\
0.2060 & 0 & 0.2595
\end{bmatrix}
\]  
(F.23)
From Equation (F.23), \( x_1 \) is the only state that participates in the mode of interest (\( \lambda_1 = -2 \)). As a result,

\[
r = x_1 \quad z = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}
\]

\[
A_{11} = -4 \quad A_{12} = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad A_{22} = \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}
\]

From Equation (F.17),

\[
M_0 = H(\lambda_1) = A_{12} (sI - A_{22})^{-1} A_{21} \big|_{s=\lambda_1} = 2
\]

Finally, the reduced order model becomes \( \dot{r}(t) = (A_{11} + M_0) r(t) = -2r(t) \). In terms of the original variables,

\[
\dot{x}_1(t) = -2x_1(t)
\]

Note that the mode of interest is preserved.

**Example 2: A pair of complex eigenvalues**

Consider the following linear system and assume that the relevant mode is the slowest one.

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 2 \\ 0 & -1 & 1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

In matrix form, eigenvalues and right- and left-eigenvectors of \( A \) are

\[
A = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix}
\]

\[
= \begin{bmatrix} -5.6274 & 0 & 0 \\ 0 & -1.6863 + j0.4211 & 0 \\ 0 & 0 & -1.6863 - j0.4211 \end{bmatrix}
\]
$$R = \begin{bmatrix}
0.9409 & -0.0601 + j0.2012 & -0.0601 - j0.2012 \\
0.0715 & 0.7615 & 0.7615 \\
-0.3309 & -0.5227 + j0.3206 & -0.5227 - j0.3206
\end{bmatrix} \quad (F.30)$$

$$L = \begin{bmatrix}
0.8902 & -0.3131 & -0.5585 \\
-0.0418 - j0.3912 & 0.6713 - j0.9326 & 0.0262 - j1.3139 \\
-0.0418 + j0.3912 & 0.6713 + j0.9326 & 0.0262 + j1.3139
\end{bmatrix} \quad (F.31)$$

Modes of interest are \(\lambda_1 = \lambda_b = -1.6863 + j0.4211\) and \(\lambda_2 = \lambda_c = -1.6863 - j0.4211\). The matrix of participation factors is

$$P = \begin{bmatrix}
0.8410 & 0.0399 & 0.0609 \\
0.0283 & 0.5318 & 0.5202 \\
0.1307 & 0.4282 & 0.4189
\end{bmatrix} \quad (F.32)$$

\(x_2\) and \(x_3\) have a high participation in the relevant modes. Thus,

$$r = \begin{bmatrix}
x_2 \\
x_3
\end{bmatrix} \quad z = x_1 \quad (F.33)$$

$$A_{11} = \begin{bmatrix}
-1 & 1 \\
-1 & -3
\end{bmatrix} \quad A_{12} = \begin{bmatrix}
0 \\
1
\end{bmatrix} \quad A_{21} = \begin{bmatrix}
1 & 2
\end{bmatrix} \quad A_{22} = -5 \quad (F.34)$$

In this case, Equation (F.17) looks like

$$M_0 [v_1, v_2] = [H(\lambda_1)v_1, H(\lambda_2)v_2].$$

However, it suffers a subtle modification in order to consider a complex pair of eigenvalues. Actually, just two equations are required. Note that four equations are obtained by separating real and imaginary parts. As \(\lambda_1 = \lambda_2^*\) and \(v_1 = v_2^*\), two out of the four equations are redundant. Thus, it is enough to consider

$$M_0 [\Re(v_1), \Im(v_1)] = [\Re(H(\lambda_1)v_1), \Im(H(\lambda_1)v_1)] \quad (F.35)$$

Here, \(\Re(\cdot)\) and \(\Im(\cdot)\) take the real part and the imaginary part of the argument, respectively. With \(v_1 = [0.7615, -0.5227 + j0.3206]^T\), \(\lambda_1 = -1.6863 + j0.4211\), then \(M_0\) is calculated as

$$H(\lambda_1) = A_{12} \left(sI - A_{22}\right)^{-1} A_{21} \big|_{s=\lambda_1}$$

$$= \begin{bmatrix}
0 & 0 \\
0.297 - j0.0377 & 0.594 - j0.0755
\end{bmatrix} \quad (F.36)$$
\[ H(\lambda_1)v_1 = \begin{bmatrix} 0 \\ -0.0601 + j0.2012 \end{bmatrix} \]  

(F.37)

From Equation (F.35),

\[ M_0 = \begin{bmatrix} 0 & 0 \\ -0.0601 & 0.2012 \end{bmatrix}^{-1} = \begin{bmatrix} 0.3517 & 0.6274 \\ -0.5227 & 0.3206 \end{bmatrix} \]  

(F.38)

Finally, the reduced order model becomes

\[ \dot{r}(t) = (A_{11} + M_0)r(t) \]  

(F.39)

\[ \Rightarrow \begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -0.6483 & -2.3726 \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} \]  

(F.40)

Note that relevant eigenvalues are preserved.

Example 3: More relevant states than relevant modes \((n > h)\)

Consider the following linear system and assume that the relevant mode is the slowest one.

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -3 & 2 \\ 1 & -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \]  

(F.41)

In matrix form, eigenvalues and right- and left-eigenvectors of \(A\) are

\[ \Lambda = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -4 + j1 & 0 \\ 0 & 0 & -4 - j1 \end{bmatrix} \]  

(F.42)

\[ R = \begin{bmatrix} 0.8729 & 0.3536 - j0.3536 & 0.3536 + j0.3536 \\ 0.4364 & 0.7071 & 0.7071 \\ 0.2182 & -0.3536 + j0.3536 & -0.3536 - j0.3536 \end{bmatrix} \]  

(F.43)
\[
L = \begin{bmatrix}
0.9165 & -0.2828 - j0.5657 & -0.2828 + j0.5657 \\
0 & 0.7071 + j0.7071 & 0.7071 - j0.7071 \\
0.9165 & -0.2828 + j0.8485 & -0.2828 - j0.8485 \\
\end{bmatrix}
\] (F.44)

The mode of interest is \(\lambda_1 = \lambda_a = -2\). The matrix of participation factors is
\[
P = \begin{bmatrix}
0.8 & 0.2150 & 0.2150 \\
0 & 0.4808 & 0.4808 \\
0.2 & 0.3041 & 0.3041 \\
\end{bmatrix}
\] (F.45)

The states that participate in the mode of interest are \(x_1\) and \(x_3\). Thus,
\[
r = \begin{bmatrix}
x_1 \\
x_3 \\
\end{bmatrix}
z = x_2
\] (F.46)

\[
A_{11} = \begin{bmatrix}
-3 & 2 \\
1 & -4 \\
\end{bmatrix}
A_{12} = \begin{bmatrix}
1 \\
-1 \\
\end{bmatrix}
A_{21} = \begin{bmatrix}
0 & 2 \\
\end{bmatrix}
A_{22} = -3
\] (F.47)

\(M_0\) is calculated following the same steps as in the previous examples.
\[
H(\lambda_1) = A_{12} \left( sI - A_{22} \right)^{-1} A_{21} \bigg|_{s=\lambda_1} = \begin{bmatrix}
0 & 2 \\
0 & -2 \\
\end{bmatrix}
\] (F.48)

\[
\Rightarrow H(\lambda_1)v_1 = \begin{bmatrix}
0.4364 \\
-0.4364 \\
\end{bmatrix}
\] (F.49)

In this case with one mode of interest and two relevant states, Equation (F.17) looks like
\[
M_0 \begin{bmatrix}
0.8729 \\
0.2182 \\
\end{bmatrix} = \begin{bmatrix}
0.4364 \\
-0.4364 \\
\end{bmatrix}
\] (F.50)

It turns out that this system of equations is undetermined and additional conditions are required to estimate \(M_0\). Typically, an objective as in optimization problems is considered. There are many different choices to determine \(M_0\). As an additional condition, use the minimum norm matrix for \(M_0\). It implies that the solution of Equation (F.17) is obtained by using the
pseudo inverse of $V_h$ as \[44\]
\[M_0 = [H(\lambda_1)v_1, ..., H(\lambda_h)v_h] V_h^+ \] \hspace{1cm} (F.51)

Here, $V_h^+$ corresponds to the pseudo inverse of $V_h$. For our particular example,
\[M_0 = H(\lambda_1)v_1V_h^+ = \begin{bmatrix} 0.4364 \\ -0.4364 \end{bmatrix} \begin{bmatrix} 0.8729 \\ 0.2182 \end{bmatrix}^+ \] \hspace{1cm} (F.52)
\[= \begin{bmatrix} 0.4364 \\ -0.4364 \end{bmatrix} \begin{bmatrix} 1.0783 & 0.2696 \end{bmatrix} = \begin{bmatrix} 0.4706 & 0.1176 \\ -0.4706 & -0.1176 \end{bmatrix} \] \hspace{1cm} (F.53)
\[\Rightarrow A_{11} + M_0 = \begin{bmatrix} -2.5294 & 2.1176 \\ 0.5294 & -4.1176 \end{bmatrix} \] \hspace{1cm} (F.54)

Then, the reduced order model becomes
\[\dot{x}_1 = -2.5294x_1 + 2.1176x_3 \] \hspace{1cm} (F.55)
\[\dot{x}_3 = 0.5294x_1 - 4.1176x_3 \] \hspace{1cm} (F.56)

The eigenvalues of the model are $-2$ and $-4.6471$. The relevant mode is preserved. The other mode has no significant value.
APPENDIX G

NEW ENGLAND TEST SYSTEM

The New England test system has 10 SGs, 39 buses, 34 lines, 12 transformers and 19 static loads. Loads and bus definition are presented in Table G.1. Bus 39 is defined as an infinite bus in dynamic simulations (the generator connected at that bus has no dynamic associated). Transformers are connected between the following buses: 2-30, 6-31, 10-32, 12-11, 12-13, 19-20, 19-33, 20-34, 22-35, 23-36, 25-37, 29-38. All transformers have a transforming ratio of 1. Their parameters are shown together with line parameters in Table G.2. With respect to SGs, a two-axis model with $X'_d = X'_q$ is considered. In addition, an IEEE Type-1 exciter and a linear speed governor with droop are considered (Figures G.1 and G.2). Their parameters, identical at every SG, are

\[
\begin{align*}
K_A &= 200 \ [pu] & T_{CH} &= 0.05 \ [s] \\
T_A &= 0.04 \ [s] & T_{SV} &= 0.1 \ [s] \\
K_E &= 1 \ [pu] & R_D &= 0.05 \ [pu] \\
T_E &= 0.7 \ [s] & K_F &= 0.08 \ [pu] \\
K_F &= 0.08 \ [pu] & T_F &= 0.8 \ [s]
\end{align*}
\]

The dynamic data of the synchronous generators are shown in Table G.3.
<table>
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<tr>
<th>Bus No</th>
<th>Bus Type</th>
<th>$P_D$ [MW]</th>
<th>$Q_D$ [Mvar]</th>
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</thead>
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<td>PQ</td>
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Table G.2: Branch data of the New England test system

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<th>To Bus</th>
<th>$R \ [pu]$</th>
<th>$X \ [pu]$</th>
<th>$B \ [pu]$</th>
<th>From Bus</th>
<th>To Bus</th>
<th>$R \ [pu]$</th>
<th>$X \ [pu]$</th>
<th>$B \ [pu]$</th>
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Figure G.1: IEEE Type-1 exciter.

Figure G.2: Linear speed governor with droop.

Table G.3: Dynamic data of the New England test system

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<th>Bus No.</th>
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<th>$V_{ref}$ [pu]</th>
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<th>$T_{q0}'$ [s]</th>
<th>$H$ [s]</th>
<th>$X_q$ [pu]</th>
<th>$X_d$ [pu]</th>
<th>$X_{d}'$ [pu]</th>
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APPENDIX H

DAMPING RATIO

For simplicity, consider that a single SG, modeled by its classical model [29], is connected to an infinite bus. Here, the SG’s internal voltage (flux linkage) is assumed constant. In addition, the effect of damper windings is considered by a damping torque $T_D = D\Delta \omega$. Thus,

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = \frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = T_m - (T_e(\delta) + T_D) \quad (H.1)$$

Linearize Equation (H.1) around the equilibrium point $\delta_0$ to obtain:

$$\frac{2H}{\omega_s} \frac{d^2\Delta \delta}{dt^2} + \frac{D}{\omega_s} \frac{d\Delta \delta}{dt} + K \Delta \delta = 0 \quad (H.2)$$

$$\Rightarrow \frac{d^2\Delta \delta}{dt^2} + \frac{\omega_s D}{2H} \frac{d\Delta \delta}{dt} + \frac{\omega_s K}{2H} \Delta \delta = 0 \quad (H.3)$$

The characteristic equation of this differential equation is $\mu^2 + \frac{\omega_s D}{2H} \mu + \frac{\omega_s K}{2H} = 0$, whose solution gives the following modes:

$$\mu_{1,2} = -\frac{\omega_s D}{4H} \pm \sqrt{\left(\frac{\omega_s D}{4H}\right)^2 - \frac{\omega_s K}{2H}} \quad (H.4)$$

As we are interested in oscillating modes, assume that the modes are complex conjugates, i.e., $\mu_{1,2} = \mu_x \pm j \mu_y$ where $\mu_x = -\frac{\omega_s D}{4H}$ and $\mu_y = \sqrt{\frac{\omega_s K}{2H} - \mu_x^2}$. Using the real and imaginary part of the modes, Equation (H.3) can be re-written as

$$\frac{d^2\Delta \delta}{dt^2} + 2(-\mu_x) \frac{d\Delta \delta}{dt} + \left(\mu_x^2 + \mu_y^2\right) \Delta \delta = 0 \quad (H.5)$$

$$\frac{d^2\Delta \delta}{dt^2} + 2\left(\frac{-\mu_x}{\sqrt{\mu_x^2 + \mu_y^2}}\right) \sqrt{\mu_x^2 + \mu_y^2} \frac{d\Delta \delta}{dt} + \left(\mu_x^2 + \mu_y^2\right) \Delta \delta = 0 \quad (H.6)$$

Define $\sigma = \frac{-\mu_x}{\sqrt{\mu_x^2 + \mu_y^2}}$ as damping ratio and $\Omega_n = \sqrt{\mu_x^2 + \mu_y^2} = \frac{\omega_s K}{2H}$ as the
undamped natural frequency. Note that if damping is neglected, Equation (H.3) would have the modes $\mu = \pm \Omega_n$. In terms of the damping ratio and undamped natural frequency, the equation of motion becomes

$$\frac{d^2 \Delta \delta}{dt^2} + 2\sigma \Omega_n \frac{d\Delta \delta}{dt} + \Omega_n^2 \Delta \delta = 0 \quad (H.7)$$

which has the following modes of oscillation: $\mu_1 = -\sigma \Omega_n + j\sqrt{1 - \sigma^2 \Omega_n}$ and $\mu_2 = -\sigma \Omega_n - j\sqrt{1 - \sigma^2 \Omega_n}$. Note that $\mu_x = -\sigma \Omega_n$ and $\mu_y = \sqrt{1 - \sigma^2 \Omega_n}$.

The solution of the equation of motion is of the form $\Delta \delta(t) = Ae^{\mu_1 t} + Be^{\mu_2 t}$ with $\Delta \delta(t = 0) = \Delta \delta_0$ and $\Delta \dot{\delta}(t = 0) = 0$ as boundary conditions. Note that $\Delta \delta(t = 0) = 0$ if the system remains in equilibrium. Thus,

$$\Delta \delta(t = 0) = A + B = \Delta \delta_0$$
$$\Delta \dot{\delta}(t = 0) = A\mu_1 + B\mu_2 = 0$$
$$\Rightarrow A = \frac{(\mu_y + j\sigma \Omega_n)}{2\mu_y} \Delta \delta_0 = \frac{(\mu_y - j\mu_x)}{2\mu_y} \Delta \delta_0$$
$$B = \frac{(\mu_y - j\sigma \Omega_n)}{2\mu_y} \Delta \delta_0 = \frac{(\mu_y + j\mu_x)}{2\mu_y} \Delta \delta_0$$

Then,

$$\Delta \delta(t) = \frac{\Delta \delta_0}{2\mu_y} e^{-\sigma \Omega_n t} \left[2(\mu_y - j\mu_x)e^{j\mu_y t} + 2(\mu_y + j\mu_x)e^{-j\mu_y t}\right]$$
$$= \frac{\Delta \delta_0}{\mu_y} e^{\sigma \Omega_n t} \left[\mu_y \cos(\mu_y t) + \mu_x \sin(\mu_y t)\right]$$
$$= \frac{\Delta \delta_0}{\mu_y} \sqrt{\mu_x^2 + \mu_y^2} e^{\sigma \Omega_n t} \left[\frac{\mu_y}{\sqrt{\mu_x^2 + \mu_y^2}} \cos(\mu_y t) + \frac{\mu_x}{\sqrt{\mu_x^2 + \mu_y^2}} \sin(\mu_y t)\right] \quad (H.8)$$

Define $\phi$ as the complementary angle of the mode $\mu_1$ so that $\cos(\phi) = \frac{\mu_y}{\sqrt{\mu_x^2 + \mu_y^2}}$ and $\sin(\phi) = \frac{\mu_x}{\sqrt{\mu_x^2 + \mu_y^2}}$. Therefore,

$$\Delta \delta(t) = \frac{\Delta \delta_0}{\cos(\phi)} e^{\sigma \Omega_n t} \left[\cos(\phi) \cos(\mu_y t) + \sin(\phi) \sin(\mu_y t)\right]$$
$$= \frac{\Delta \delta_0}{\cos(\phi)} e^{\sigma \Omega_n t} \cos(\mu_y t - \phi)$$
$$= \frac{\Delta \delta_0}{\cos(\phi)} e^{-\sigma \Omega_n t} \cos(\mu_y t - \phi) \quad (H.9)$$

Equation (H.9) is an approximated solution of the motion of the SG.
Table H.1: Attenuation of oscillating modes for different damping ratios and different times

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<th>$\sigma$</th>
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<th>$3T$</th>
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<td>88.19%</td>
<td>82.82%</td>
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<td>82.81%</td>
<td>68.58%</td>
<td>56.79%</td>
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<td>53.31%</td>
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<td>7%</td>
<td>64.35%</td>
<td>41.40%</td>
<td>26.64%</td>
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around an equilibrium point. Now, consider a stable equilibrium point. When the system is perturbed, i.e., $\Delta \delta_0 \neq 0$, the amplitude of oscillations are reduced to $e^{-\sigma \sqrt{1-\sigma^2} \mu_y t} \times 100\%$ of its initial value at time $t$. The oscillating period is $T = \frac{2\pi}{\mu_y}$. Thus, the reduction of oscillations at time $T$, $2T$ and $3T$ for different damping ratios is as shown in Table H.1. Thus, when the damping ratio is 5%, the oscillation is damped to about 39% of its initial value in three oscillation periods. In general, modes are not critical if they have at least a damping ratio of 5%.
APPENDIX I

WIND FARM DATA

Two wind farms are considered: Wind Farm A has 20 WTGs and Wind Farm B has 50 WTGs. The diagrams are shown in Figures I.1 and I.2. The wind farm’s network parameters are presented in Tables I.1 and I.2. Data is partially obtained from [36]. For simplicity, the step-up transformers of WTGs are neglected. Therefore, the WTG’s parameters presented in this appendix are already referred to the high voltage side of the omitted WTGs’ transformers (23 [kV]). In addition, the parameters of all WTGs are assumed to be identical and are as follows:

\[
\begin{align*}
X_s &= 3.5547 \text{ [pu]} & S_{WTG} &= 5 \text{ [MVA]} \\
X_r &= 3.5859 \text{ [pu]} & Q_{ref} &= 0 \text{ [Mvar]} \\
X_m &= 3.5092 \text{ [pu]} & B &= 1.321 \times 10^{-3} \text{ [pu s}^3/m^3]\]
\begin{align*}
R_s &= 0.1015 \text{ [pu]} & C &= 1.066 \times 10^{-8} \text{ [pu s}^3]\]
R_r &= 0.0880 \text{ [pu]} & D &= 0.1667 \text{ [-]} \\
K_{P1} &= 1 \text{ [pu]} & K_{I1} &= 5 \text{ [pu]} \\
K_{P2} &= 1 \text{ [pu]} & K_{I2} &= 5 \text{ [pu]} \\
K_{P3} &= 1 \text{ [pu]} & K_{I3} &= 5 \text{ [pu]} \\
K_{P4} &= 1 \text{ [pu]} & K_{I4} &= 5 \text{ [pu]} \\
H &= 4 \text{ [s]}
\end{align*}
\]

Note that if the system’s power base, \(S_b\), is different than \(S_{WTG}\), then \(X_s, X_r, X_m, R_s, R_r, K_{I2}, K_{I4}, K_{P2} \text{ and } K_{P4}\) have to be scaled as

\[
Parameter^{new} \text{ [pu]} = Parameter^{old} \text{ [pu]} \frac{S_b}{S_{WTG}}
\]
In addition, the H-constant of inertia and the turbine's parameters $B$ and $C$ have to be scaled too as

$$H^{new}[s] = H^{old}[s] \frac{S_{WTG}}{S_b}$$ \hspace{1cm} (I.3)

$$B^{new}[pu \ s^3/m^3] = B^{old}[pu \ s^3/m^3] \frac{S_{WTG}}{S_b}$$ \hspace{1cm} (I.4)

$$C^{new}[pu \ s^3] = C^{old}[pu \ s^3] \frac{S_{WTG}}{S_b}$$ \hspace{1cm} (I.5)
Figure I.2: Wind Farm B’s diagram.
Table I.1: Parameters of the Wind Farm A’s network

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<th>From bus</th>
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<th>$B$ [pu]</th>
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Table I.2: Parameters of the Wind Farm B’s network

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REFERENCES


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AUTHOR’S BIOGRAPHY

Hector A. Pulgar-Painemal received a B.S. degree and an M.S. degree in Electrical Engineering from the University of Concepcion, Chile, in 2001 and 2003, respectively. During 1999-2000, he was a teaching assistant at the University of Concepcion. In 2001, he was the recipient of the University of Concepcion Award for best student in his class. During 2001-2002, he was hired for the teaching staff at the Federico Santa Maria Technical University, Chile. In 2003, he was promoted to the position of Academic Instructor. Since then, he has taught over five courses, participated in two research projects and guided several student final projects in the Electrical Engineering Program. He entered the Ph.D. program in the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign in 2006 with a Fulbright Fellowship and the support of the Federico Santa Maria Technical University. After finishing his Ph.D. studies, Mr. Pulgar-Painemal plans to return to the Federico Santa Maria Technical University as a professor. His research activities are in the areas of power system dynamics, power system operation, wind power generation, and power system simulations.