MIXED CONVECTION FLOW AND HEAT TRANSFER IN A LID-DRIVEN CAVITY SUBJECTED TO NANOFLUID: EFFECT OF TEMPERATURE, CONCENTRATION AND CAVITY INCLINATION ANGLES

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In this work, mixed convection in a lid-driven cavity with an inside heated obstacle at various cavity inclination angles with new formulation of variable properties was studied numerically. The bottom and vertical walls are kept insulated, whereas the top moving wall is maintained at a low temperature $T_c$. An obstacle of a relatively higher temperature $T_h$ is located on the bottom wall of the cavity. The governing equations are discretized using the finite volume method and SIMPLER algorithm. Using a developed code, we accomplished a parametric study and analyzed the influence of important parameters, such as the Richardson number, solid volume fraction of $\text{Al}_2\text{O}_3$ nanoparticles, cavity inclination angles, height of a hot obstacle, and the nanofluid temperature on the thermal behavior and flow characteristics.

KEY WORDS: nano scale particles, heat transfer, mixed convection, lid-driven cavity

1. INTRODUCTION

Widespread use of mixed convection phenomena (which combine both natural and forced convection) in engineering and industry has attracted the attention of many researchers. In recent years, some interesting researches have particularly been done by Talebi et al. (2010), Abunada and Chamkha (2010), Mahmoodi (2011a), Ghasemi and Aminossadati (2010), and Arefmanesh and Mahmoodi (2011).

Recently, Abbasian et al. (2012) investigated a mixed convection flow in a lid-driven square cavity. A Cu–water nanofluid was inside the cavity whose horizontal walls were adiabatic, while its sidewalls were heated sinusoidally. They proved that decreasing the Richardson number and increasing the volume fraction of nanoparticles...
causes an increase in the rate of heat transfer. Moreover, they showed that with increase in the Richardson number at a constant Reynolds number, the rate of heat transfer increases.

Heat transfer can be improved by adding nanoparticles into the cavity. This statement was expressed by many researchers, such as Arefmanesh et al. (2012) who numerically studied a natural convection fluid flow and heat transfer in annuli filled with a TiO$_2$–water nanofluid. The annuli were included into two differentially heated square ducts, with the inner duct being kept at a high constant temperature, while the outer one maintained at a low constant temperature. Their results showed that with increase in the volume fraction of the nanoparticles the average Nusselt number also increases.

\begin{table}
\centering
\begin{tabular}{|l|l|}
\hline
NOMENCLATURE & \textit{X}, \textit{Y} dimensionless Cartesian coordinates \\
\hline
\textit{c}_p & specific heat, J-kg$^{-1}$-K$^{-1}$ \\
\hline
Gr & Grashof number \\
\hline
\textit{g} & gravitational acceleration, m-s$^{-2}$ \\
\hline
\textit{h} & heat transfer coefficient, W-m$^{-2}$-K$^{-1}$ \\
\hline
\textit{L} & enclosure length, m \\
\hline
\textit{k} & thermal conductivity, W-m$^{-1}$-K$^{-1}$ \\
\hline
\textit{Nu} & Nusselt number \\
\hline
\textit{p} & pressure, N-m$^{-2}$ \\
\hline
\textit{P} & dimensionless pressure \\
\hline
\textit{Pr} & Prandtl number \\
\hline
\textit{q} & heat flux, W-m$^{-2}$ \\
\hline
\textit{Re} & Reynolds number \\
\hline
\textit{Ri} & Richardson number \\
\hline
\textit{T} & dimensional temperature, K \\
\hline
\textit{u}, \textit{v} & dimensional velocity components in \textit{x} and \textit{y} directions, m-s$^{-1}$ \\
\hline
\textit{U}, \textit{V} & dimensionless velocity components in \textit{x} and \textit{y} directions \\
\hline
\textit{U}_0 & lid velocity \\
\hline
\textit{x}, \textit{y} & dimensional Cartesian coordinates, m \\
\hline
\end{tabular}
\end{table}

Greek symbols

\begin{itemize}
\item $\alpha$ thermal diffusivity, m$^2$s$^{-1}$
\item $\beta$ thermal expansion coefficient, K$^{-1}$
\item $\theta$ dimensionless temperature
\item $\mu$ dynamic viscosity, kg-m$^{-1}$s$^{-1}$
\item $\nu$ kinematic viscosity, m$^2$s$^{-1}$
\item $\rho$ density, kg-m$^{-3}$
\item $\varphi$ volume fraction of nanoparticles
\item $\gamma$ cavity inclination angle
\end{itemize}

Subscripts

\begin{itemize}
\item \textit{c} cold
\item \textit{eff} effective
\item \textit{f} fluid
\item \textit{h} hot
\item \textit{nf} nanofluid
\item \textit{s} solid particles
\item \textit{w} wall
\end{itemize}
Another work done by Nikfar and Mahmoodi (2012) also approved the above statement. They studied natural convection in a square cavity filled with an Al₂O₃–water nanofluid. The horizontal walls of the cavity were insulated, while the left and right wavy sidewalls of the cavity were maintained at high and low constant temperatures. They demonstrated that an increase in the volume fraction of the nanoparticles leads to an increase in the average Nusselt number of the hot wall.

The effect of the existence of an obstacle within the cavity is one of the interesting challenges for researchers. Recently, a free-convective fluid flow and heat transfer were investigated numerically by Mahmoodi and Mazrouei (2012). They studied a Cu–water nanofluid around adiabatic square bodies at the center of a square cavity. They showed that for most Rayleigh numbers the Nusselt number increases with increase in the volume fraction of nanoparticles. They also showed that at low Rayleigh numbers, the rate of heat transfer decreases on increase in the size of the adiabatic square body, and that the opposite is true at high Rayleigh numbers.

The effect of the heater on the cavity has also been considered in recent years. For instance, an inside thin heater with varying location and length was located inside a square cavity. Different water-based nanofluids were verified numerically by Mahmoodi (2011b). Vertical walls of the cavity had a low constant temperature, while its horizontal walls were insulated. The results obtained showed that at high Rayleigh numbers, despite the low Rayleigh numbers, the position of the heater does not affect the rate of heat transfer. Also, he concluded that at low Rayleigh numbers the rate of heat transfer is not affected by the type of nanofluids, while at high Rayleigh numbers, the type of a nanofluid is more effective in improving the heat transfer rate.

In another work, Mazrouei et al. (2012) considered a square cavity with a heat source on the bottom wall with moving cold sidewalls and insulated top wall. They verified the influence of variable properties on mixed convection within the cavity. They observed that as the Rayleigh number was kept constant, the addition of nanoparticles improved the heat transfer rate, when the heat source was located in the middle of the bottom wall. Moreover, they showed that the rate of heat transfer increased when the heat source moved toward the sidewall. In this study, a numerical simulation of an inclined square cavity containing a heated obstacle was used to investigate the influence of variation of some parameter such as the Richardson number, solid volume fraction, cavity inclination angles and so on on the thermal behavior and flow characteristics.

In this study, a nanofluid-filled lid-driven cavity containing a hot obstacle has been analyzed at various cavity inclination angles with new formulation of variable properties. These new models are used to evaluate the thermal conductivity and dynamic viscosity of the nanofluid inside the cavity. Also, the effect of changes in the pertinent parameters such as the Richardson number, solid volume fraction of Al₂O₃ nanoparticles, cavity inclination angles, height of a hot obstacle, and the nanofluid temperature on heat transfer performance and fluid flow are investigated.
2. PHYSICAL MODEL AND GOVERNING EQUATIONS

A schematic view of the inclined nanofluid-filled lid-driven cavity considered in the present paper is displayed in Fig. 1. The height and width of the square cavity are denoted by $L$. The bottom and vertical walls are kept insulated, whereas the top moving wall is maintained at a low temperature $T_c$. An obstacle of a relatively higher temperature $T_h$ is located on the bottom wall of the cavity. The length and location of the hot obstacle are denoted by $d$ and $h$, respectively.

The nanofluid in the enclosure is Newtonian, incompressible, and laminar. In addition, it is assumed that both the fluid phase and nanoparticles are in the thermal equilibrium state and they flow with the same velocity. The density variation in the body force term of the momentum equation is satisfied by Boussinesq’s approximation. The thermophysical properties of nanoparticles and water as of the base fluid at $T = 25^\circ C$ are presented in Table 1.

The thermal conductivity and the viscosity of the nanofluid are taken into consideration as variable properties; both of them change with the volume fraction and temperature of nanoparticles. With the above assumptions, the system of governing equations is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u + \frac{(\rho \beta)_{nf}}{\rho_{nf}} g \Delta T \sin (\gamma), \quad (2)$$

FIG. 1: A schematic view of the cavity considered in the present study
TABLE 1: Thermophysical properties of water and nanoparticles at $T = 25^\circ$C

<table>
<thead>
<tr>
<th>Solid (Al$_2$O$_3$)</th>
<th>Fluid phase (water)</th>
<th>Physical properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>765</td>
<td>4179</td>
<td>$c_p$ (J/kg·K)</td>
</tr>
<tr>
<td>3970</td>
<td>997.1</td>
<td>$\rho$ (kg/m$^3$)</td>
</tr>
<tr>
<td>25</td>
<td>0.6</td>
<td>$K$ (W·m$^{-1}$·K$^{-1}$)</td>
</tr>
<tr>
<td>0.85</td>
<td>21.</td>
<td>$\beta \times 10^{-5}$ (1/K)</td>
</tr>
<tr>
<td>........</td>
<td>8.9</td>
<td>$\mu \times 10^{-4}$ (kg/m·s)</td>
</tr>
<tr>
<td>47</td>
<td>0.384</td>
<td>$d_p$ (nm)</td>
</tr>
</tbody>
</table>

\[
\frac{u}{\hat{c}_x} + \nu \frac{\partial u}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 u + \frac{(p\beta)_{nf}}{\rho_{nf}} g \Delta T \cos (\gamma), \tag{3}
\]

and

\[
\frac{u}{\hat{c}_x} + \nu \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T. \tag{4}
\]

The dimensionless parameters may be presented as

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{\nu}{u_0}, \quad U = \frac{u}{u_0},
\]

\[
\Delta T = T_h - T_c, \quad \theta = \frac{T - T_c}{\Delta T}, \quad P = \frac{p}{\rho_{nf} u_0^2}.
\]

Hence,

\[
Re = \frac{\rho_f u_0 L}{\mu_f}, \quad \text{Ri} = \frac{Ra}{Pr \ Re^2}, \quad \text{Ra} = \frac{g \beta_f \Delta T L^3}{\nu_f \alpha_f}, \quad \text{Pr} = \frac{\nu_f}{\alpha_f}.
\]

The above governing equations (1) to (4) in the dimensionless form are

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{7}
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 U + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \sin (\gamma), \tag{8}
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 V + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \cos (\gamma). \tag{9}
\]
and

\[ \frac{U}{X} \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta. \]  

(10)

2.1 Thermal Diffusivity and Effective Density

The thermal diffusivity and effective density of the nanofluid are

\[ \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \]  

(11)

\[ \rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f. \]  

(12)

2.2 Heat Capacity and Thermal Expansion Coefficient

The heat capacity and thermal expansion coefficient of the nanofluid are

\[ (\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_f, \]  

(13)

\[ (\rho \beta)_{nf} = \phi (\rho \beta)_s + (1 - \phi) (\rho \beta)_f. \]  

(14)

2.3 Viscosity

The effective viscosity of the nanofluid was calculated by

\[ \mu_{eff} = \mu_f (1 + 2.5 \phi) \left[ 1 + \eta \left( \frac{d_p}{L} \right)^{2 \epsilon} \varphi^{\frac{2\epsilon}{3}} (\epsilon + 1) \right]. \]  

(15)

This well-validated model is presented by Jang et al. (2007) for a fluid containing a dilute suspension of small rigid spherical particles and it accounts for the slip mechanism in nanofluids. The empirical constants \( \epsilon \) and \( \eta \) are 0.25 and 280 for Al\(_2\)O\(_3\), respectively.

It is worth mentioning that the viscosity of the base fluid (water) is considered to vary with temperature, and the flow equation is used to evaluate the viscosity of water:

\[ \mu_{H_2O} = (1.2723 \times T_{rc}^5 - 8.736 \times T_{rc}^4 + 33.708 \times T_{rc}^3 - 246.6 \times T_{rc}^2 \]  

\[ + 518.7 \times T_{rc} + 1153.9) \times 10^6, \]  

(16)

where \( T_{rc} = \log (T - 273). \)
2.4 Dimensionless Stagnant Thermal Conductivity

The effective thermal conductivity of the nanoparticles in the liquid is calculated by the Hamilton and Crosser (H–C) model (Hamilton and Crosser, 1962)

\[
\frac{k_{\text{stationary}}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}.
\]

(17)

2.5 Total Dimensionless Thermal Conductivity of Nanofluids

\[
\frac{k_{nf}}{k_f} = \frac{k_{\text{stationary}}}{k_f} + \frac{k_c}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}
\]

\[
+ c \frac{\text{Nu}_p d_f (2 - D_f) D_f}{\text{Pr} (1 - D_f)^2} \left[ \left( \frac{d_{\text{max}}}{d_{\text{min}}} \right)^{1-D_f} - 1 \right] \left( \frac{d_{\text{max}}}{d_{\text{min}}} \right)^{2-D_f} - 1 \frac{d_p}{d_{\text{p},\text{min}}}. \]

This model was proposed by Xu et al. (2006) and it has been chosen for this study to describe the thermal conductivity of nanofluids. The quantity \(c\) is an empirical constant (e.g., \(c = 85\) for deionized water and \(c = 280\) for ethylene glycol) which is independent of the type of nanoparticles; \(\text{Nu}_p\) is the Nusselt number for the liquid flowing around a spherical particle and equal to two for a single particle in this work. The fluid molecular diameter is \(d_f = 4.5 \times 10^{-10} \text{ m}\) for water in the present study. The fractal dimension \(D_f\) is determined by

\[
D_f = 2 - \frac{\ln \varphi}{\ln \left( \frac{d_{\text{p},\text{min}}}{d_{\text{p},\text{max}}} \right)},
\]

where \(d_{\text{p},\text{max}}\) and \(d_{\text{p},\text{min}}\) are the maximum and minimum diameters of nanoparticles, respectively. The ratio of the minimum to maximum diameters of nanoparticles \(d_{\text{p},\text{min}}/d_{\text{p},\text{max}}\) is \(R:\)

\[
d_{\text{p},\text{max}} = d_p \frac{D_f}{D_f - 1} \left( \frac{d_{\text{p},\text{min}}}{d_{\text{p},\text{max}}} \right)^{-1},
\]

\[
d_{\text{p},\text{min}} = d_p \frac{D_f - 1}{D_f}.
\]
3. NUMERICAL APPROACH

The governing equations for continuity, momentum, and energy conservation associated with the boundary conditions in this investigation were calculated numerically based on the finite volume method and associated staggered grid system, using the FORTRAN computer code. The SIMPLER algorithm is used to solve the coupled system of governing equations. The convection terms is approximated by a hybrid scheme which is conducive to a stable solution. In addition, a second-order central differencing scheme is utilized for the diffusion terms. The algebraic system resulting from numerical discretization was calculated utilizing TDMA applied in a line going through all volumes in the computational domain. To verify the grid independence, a numerical procedure was applied to nine different mesh sizes. The average Nu number of the hot body wall at $w = 0.2$, $\text{Ri} = 1$, $h = 0.1$, $\gamma = 30^\circ$, and $\varphi = 0.03$ is obtained for each grid size, as shown in Fig. 2.

As can be observed, the $91 \times 91$ uniform grid size yields the required accuracy. It was hence applied in this work as presented in the next section.

To ensure the accuracy and validity of this new model, we analyze a square cavity filled with a base fluid with $\text{Pr} = 0.7$ and different $\text{Ra}$ numbers. Table 2 compares the results obtained by our new model with the values available in the literature. The quantitative comparisons for the average Nusselt numbers indicate an excellent agreement between them.

![FIG. 2: The mesh used](image-url)
TABLE 2: Code validation of the comparison between the results of the present study and other research works

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b) $Ra = 10^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>16.052</td>
<td>16.158</td>
<td>16.1439</td>
<td>15.995</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.817</td>
<td>0.819</td>
<td>0.822</td>
<td>0.814</td>
</tr>
<tr>
<td>$v_{max}$</td>
<td>19.528</td>
<td>19.648</td>
<td>19.665</td>
<td>18.894</td>
</tr>
<tr>
<td>$X$</td>
<td>0.110</td>
<td>0.112</td>
<td>0.110</td>
<td>0.103</td>
</tr>
<tr>
<td>$Nu_{ave}$</td>
<td>2.215</td>
<td>2.243</td>
<td>2.195</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td>(c) $Ra = 10^5$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>36.812</td>
<td>36.732</td>
<td>34.30</td>
<td>37.144</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.856</td>
<td>0.858</td>
<td>0.856</td>
<td>0.855</td>
</tr>
<tr>
<td>$v_{max}$</td>
<td>68.791</td>
<td>68.288</td>
<td>68.7646</td>
<td>68.914</td>
</tr>
<tr>
<td>$X$</td>
<td>0.062</td>
<td>0.063</td>
<td>0.05935</td>
<td>0.061</td>
</tr>
<tr>
<td>$Nu_{ave}$</td>
<td>4.517</td>
<td>4.511</td>
<td>4.450</td>
<td>4.964</td>
</tr>
<tr>
<td></td>
<td>(d) $Ra = 10^6$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>66.445</td>
<td>66.46987</td>
<td>65.5866</td>
<td>66.42</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.873</td>
<td>0.86851</td>
<td>0.839</td>
<td>0.897</td>
</tr>
<tr>
<td>$v_{max}$</td>
<td>221.748</td>
<td>222.33950</td>
<td>219.7361</td>
<td>226.4</td>
</tr>
<tr>
<td>$X$</td>
<td>0.0398</td>
<td>0.03804</td>
<td>0.04237</td>
<td>0.0206</td>
</tr>
<tr>
<td>$Nu_{ave}$</td>
<td>8.795</td>
<td>8.757933</td>
<td>8.803</td>
<td>10.39</td>
</tr>
</tbody>
</table>

FIG. 3: Comparison between the current study and that of Talebi et al. (2010) at $Re = 1$, $Ra = 1.47$, and $\phi = 0.03$
Also, the proposed numerical scheme is validated by comparing the present code results with the numerical simulation published by Talebi et al. (2010) (Fig. 3). It is clear that the present code is in excellent agreement with another work reported in the literature, as is seen from Fig. 3.

4. RESULTS AND DISCUSSION

In this study, an inclined square cavity filled with a nanofluid and having a hot four-sided obstacle has been simulated numerically based on the finite volume method. The effects of four substantial parameters such as the Richardson number, solid volume fraction, cavity inclination angles, and the fluid temperature on the thermal behavior and flow characteristics have been studied.

Figure 4 exhibits streamlines and isotherms for the cavity positioned at different angles at Re = 100, $\varphi = 0.05$, $T = 300$, $d_p = 47$ nm, and $h = 0.1L$. The flow pattern in the case of the horizontal position of the cavity shows the formation of a primary clockwise cell which occupies almost all the cavity space. The main reason for the formation of this strong central cell is the codirected action of the buoyancy force occurring at the temperature difference and of the shear force caused by the movement of the upper lid which amplify each other because of their codirectionality. As the cavity inclination increases to $30^\circ$, the buoyancy and shear forces lose their perfect codirectionality and the flow pattern in the cavity changes. In such a state, two small vortices appear as a result of the force of the upper moving lid and the region near the lid itself, while the intensity of the main cell reduces a little. As a result of the dominance of natural convection over forced convection in the Richardson number range studied, the central cell resulted from the buoyancy force is stronger at all positions than the upper vortices caused by the shear force from the upper lid. By increasing the cavity inclination angle, we oppose the actions of the shear and buoyancy forces are and hence — as it is obvious from the figure — the intensity and strength of the central cell decrease and the upper small vortices are amplified. The isotherm lines exhibit greater intensity in the areas close to the hot walls.

As can be seen from the figure, when we change the angle from zero to $30^\circ$, the intensity of these lines is reduced in the area close to the walls of the hot obstacle. With reduction in the intensity of these lines, the temperature gradient reduces. Therefore, it is expected that the heat transfer rate is reduced through changing the inclination angle. A further increase in the cavity inclination angle does not cause meaningful changes in the isotherms and accordingly a viable estimation of the heat transfer rate and of a decrease or increase in it at these angles cannot be made based on these figures.

Changes in the streamlines and temperature at $Ri = 0.01$, $\varphi = 0.05$, $\gamma = 60^\circ$, $d_p = 47$ nm, and $h = 0.1L$ are shown in Fig. 5 for different temperatures of the nanofluid inside the cavity. These changes are analyzed for three different temperatures of 300,
FIG. 4: Streamlines and isotherms at different angles of the cavity position at Re = 100, φ = 0.05, T = 300, dp = 47 nm, and h = 0.1L
FIG. 5: Streamlines and isotherms at different temperatures of the nanoufluid at \( \text{Re} = 0.01, \gamma = 60^\circ, \varphi = 0.05, d_p = 47 \text{ nm}, \) and \( h = 0.1L \)
320, and 340°C. As can be seen in this figure, the flow pattern displays the formation of a clockwise central cell with a circular core caused by the shear force from the upper lid and two small vortices caused by the buoyancy force on the sides of the obstacle. Changes in the nanofluid temperature do not cause a meaningful change in the flow pattern. The temperature lines also exhibit high intensity in the areas close to the hot obstacle walls. The very high temperature gradient resulted from the intensity of these lines indicates the dominance of forced convection over natural convection. As it can be seen from the figure, on increase in the temperature of the nanofluid, the intensity of these lines decreases around the wall and, as a result, the temperature gradient decreases. Accordingly, it could be expected that the heat transfer rate and the average Nusselt number in the cavity will decrease significantly with temperature that increases in this parametric interval.

Figure 6 shows changes in the flow and temperature lines at different Richardson numbers and at $\phi = 0.05$, $\gamma = 30^\circ$, $T = 300$, $d_p = 47$ nm, and $h = 0.3L$. At $Ri = 0.01$ and with perfect excess of forced convection over natural convection, the streamlines indicate the formation of a strong cell in the upper and central areas of the cavity. This cell is caused by the force resulted from the movement of the upper lid. The existence of two weak small vortices on the sides of the obstacle is an indicator of a minor buoyancy force caused by the temperature difference. By raising the Richardson number and amplifying buoyancy over the shear force, we can move the central clockwise cell downward and intensify it. The composition of the big central cell in the cavity indicates relative codirectionality of the buoyancy and shear forces at this particular angle. Also, the isotherms show high intensity at $Ri = 0.01$ particularly in the areas close to the hot obstacle walls. The high intensity of these lines indicates a high temperature gradient and a significant heat transfer rate at these points. On increase in the Richardson number and in the case of the excess of natural convection over forced convection, the intensity of lines around the walls decreases significantly. Therefore it is expected that by increasing the Richardson number in these parameter intervals, we can significantly decrease the heat transfer rate.

Figure 7 shows the variation of the Nusselt number with the Richardson number at different angles and at $\phi = 0.05$, $T = 300$, $d_p = 47$ nm, and $h = 0.1L$. As expected, the codirectionality of the buoyancy and shear forces in the cavity in the horizontal position improves heat transfer as compared to other degrees of inclination. The thermal behavior of the fluid does not abide by any particular regulation at different angles, but at most values of the Richardson number, an increase in the inclination angle of the cavity results in a decrease of the heat transfer rate. Also, as already stated during the analysis of Fig. 5, with increase in the Richardson number, the value of the Nusselt number decreases and the heat transfer rate decreases, as a result.

Figure 8 shows the Nusselt number vs. the Richardson number for different temperatures of the nanofluid at $\phi = 0.05$, $\gamma = 30^\circ$, $d_p = 47$ nm, and $h = 0.1L$. It is clear from this diagram that an increase in the temperature of the nanofluid causes a de-
FIG. 6: Streamlines and isotherms at different Richardson numbers at $\varphi = 0.05$, $\gamma = 30^\circ$, $T = 300$, $d_p = 47$ nm, and $h = 0.3L$
increase in the Nusselt number and, thus, in the heat transfer rate. Additionally, in this parametric interval, the Richardson number increases too and causes a decrease in the heat transfer rate.

Nusselt numbers for a square cavity in different volume fractions of the nanofluid and base fluid at $\gamma = 30^\circ$, $d_p = 47$ nm, $T = 300$, and $h = 0.1L$ are presented in Fig. 9. As can be seen, an addition of nanoparticles to water and increase in the solid volume

![Graph](image1.png)

**FIG. 7:** Nusselt number vs. the Richardson number at different inclination angles of the cavity

![Graph](image2.png)

**FIG. 8:** Nusselt number vs. the Richardson number for various temperatures of the nanofluid

![Graph](image3.png)

**FIG. 9:** Nusselt number vs. the Richardson number at different solid volume fractions of the nanofluid
fraction of nanoparticles in the fluid can result in an increase in the Nusselt number and in the heat transfer rate. Too, there are much more differences in the heat transfer rates at lower Richardson numbers compared to higher ones.

Figure 10 shows the variation of the Nusselt number at different heights of the hot obstacle and different Richardson numbers with $\varphi = 0.05$, $\gamma = 90^\circ$, $d_p = 47$ nm, and $T = 300$. The existence of the difference in the heat transfer rates at different heights of the obstacle indicates that adjustments of the obstacle height can be utilized as a mechanism aimed at controlling the heat transfer rate, cooling and heating inside the cavity.

As the diagram shows, an increase in the height of the hot obstacle results in a decrease in the heat transfer rate. At lower Richardson numbers, an increase and decrease in the height has a more substantial effect on the thermal behavior of the inside of the cavity.

5. CONCLUSIONS

In this study, the problem of combination of natural and forced convection in a cavity filled with a nanofluid (Al$_2$O$_3$–water) of variable properties and having inside a hot obstacle was performed numerically based on the finite volume method. A parametric study was done and effects of the Richardson number, cavity inclination angles, volume fraction of the Al$_2$O$_3$ nanoparticles, and of the temperature of the nanofluid on the heat transfer performance inside the cavity were studied and the following results were obtained:

**FIG. 10**: Nusselt number vs. the Richardson number at different inclination angles of the cavity
1) adding nanoparticles to the base fluid and increasing the solid volume fraction increase the Nusselt number and heat transfer rate in all ranges of the parameters;
2) for all cases considered, where the volume fraction of the nanoparticles is kept constant, the rate of heat transfer decreases on increase in the Richardson number;
3) increasing the height of the cavity reduces the temperature gradient and consequently decreases the heat transfer rate;
4) when the Richardson number is kept constant, the average Nusselt number increases on decrease in the temperature of the nanofluid.

REFERENCES

