EFFECT OF RAYLEIGH NUMBER ON MIXED CONVECTION FLOW AND HEAT TRANSFER IN AN INCLINED TWO-SIDED LID-DRIVEN CAVITY SUBJECTED TO A VARIABLE PROPERTIES NANO-FLUID

Mohammad Hemmat Esfe, Mohammad Akbari, Seyed Hadi Rostamian
Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran

ABSTRACT: To analyze the heat transfer and fluid flow of mixed convection through nano-fluid in an inclined square cavity having two hot obstacles, a numerical method based on the finite volume approach is applied. In this investigation Al₂O₃-water at T=300 and diameter of nanoparticle equal to 40, have been used as working fluid. The study has been executed for the Rayleigh number between 10⁴ and 10⁵. Results show that increasing Ra number leads to heat transfer increases.


INTRODUCTION
Nano-fluids are a suspension of nano-sized solid particles in a base fluid ethylene glycol, water or propylene glycol. The first to coin the “nano-fluids” for these fluids with higher thermal properties was Choi. (1995). Existence of high thermal conductivity metallic nanoparticles in base fluid increases the thermal conductivity of such mixtures; hence conventional fluids have a rather low thermal conductivity with respect to nano-fluids. Therefore, different types of nano-fluids are used to enhance the rate of heat transfer in many practical engineering applications (Choi et al., 2004; Godson et al., 2010; Sarkar, 2011).

Numerical simulation to examine the effects of inclination angle on free convection in enclosures subjected to a nano-fluid has been performed by Abu-Nada and Oztop. (2009). Recently, a research was carried out by Mahmoudi et al. (2010) focusing on free convection around a heat source in a square cavity. Mixed convection is a type of convection which combination of both natural and forced convection, is a very important and complex heat transfer mechanism. Mixed convection flow with lid driven effect has many applications in industry and engineering such as chemical processing equipment, drying technologies, cooling of the electronic devices, lubrication technologies, etc. There are several investigators on the mixed convection heat transfer in single or two-sided lid-driven cavities due to importance of this phenomenon. Shahi et al. (2010) has made a numerical investigation of the mixed convection in a ventilated square cavity. They considered effects of Richardson number and solid volume fraction of nano particle, and found that increase in solid volume fraction of nano particle led to increase in the average Nusselt number of the heat source.

A numerical study of the mixed convection flow and heat transfer performance of nano-fluid in a lid-driven cavity has been executed by Muthtamilselvan et al. (2010). Recently, Arefmanesh and Mahmoodi, (2011) carried out a numerical simulation to investigate uncertainties effects of dynamic viscosity models for Al₂O₃-water nano-fluid on mixed convection in a square cavity. In this study, for the first time mixed convection in an inclined lid-driven cavity containing two heated obstacles subjected to Al₂O₃-water nano-fluid are examined. Another studies have been executed on different aspect of mixed convection by Fereidoon et al., (2013); Zarei et al., (2013); Abbasian Arani et al., (2012).

MATHEMATICAL MODEL
Figure 1 shows a two-dimensional square lid-driven inclined cavity considered for the present study with physical dimensions. The cavity is subjected to nano-fluid. Two hot square bodies are located at the bottom wall of the square enclosure. The height and the width of these bodies are denoted by h and d.

Figure 1: Schematic diagram of lid-driven cavity.
The cavity is filled with a suspension of $\text{Al}_2\text{O}_3$ nanoparticles in water that the nanoparticles and the base fluid are in thermal equilibrium and there is no slip between them. The thermo-physical properties of nanoparticles and the water as the base fluid at $T = 25^\circ\text{C}$ are presented in Table 1.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (Water)</th>
<th>Solid ($\text{Al}_2\text{O}_3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>K (W m$^{-1}$K$^{-1}$)</td>
<td>0.6</td>
<td>25</td>
</tr>
<tr>
<td>$\beta \times 10^{-5}$ (1/K)</td>
<td>21.</td>
<td>0.85</td>
</tr>
<tr>
<td>$\mu \times 10^{-4}$ (Kg/ms)</td>
<td>8.9</td>
<td>.....</td>
</tr>
</tbody>
</table>

The thermal conductivity and the viscosity of the nano-fluid are taken into consideration as variable properties; both of them change with volume fraction and temperature of nanoparticles. Under the above assumptions, the system of governing equations is:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
$$

(1)

$$
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u + \left(\frac{\rho \beta_{nf}}{\rho_{nf}}\right) g \Delta T \sin(\gamma)
$$

(2)

$$
u \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 v + \left(\frac{\rho \beta_{nf}}{\rho_{nf}}\right) g \Delta T \cos(\gamma)
$$

(3)

And

$$
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T.
$$

(4)

The dimensionless parameters may be presented as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{V}{u_0}, \quad U = \frac{U}{u_0}
$$

(5)

$$\Delta T = T_h - T_c, \quad \theta = \frac{T - T_c}{\Delta T_f}, \quad P = \frac{p}{\rho_{nf} u_0^2}
$$

Hence,

$$\text{Re} = \frac{\rho_0 u_0 L}{\mu_f}, \quad \text{Ri} = \frac{\text{Ra}}{\text{Pr} \text{Re}^\gamma}, \quad \text{Ra} = \frac{\beta \Delta T L^3}{\nu \alpha_f}, \quad \text{Pr} = \frac{\nu_f}{\alpha_f}
$$

(6)

The dimensionless form of the above governing equations (1) to (4) become

$$
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
$$

(7)

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\nu_f} \nabla^2 U + \frac{Ri}{\text{Pr}} \frac{\beta_{nf}}{\beta_f} \Delta \theta \sin(\gamma)
$$

(8)

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\nu_f} \nabla^2 V + \frac{Ri}{\text{Pr}} \frac{\beta_{nf}}{\beta_f} \Delta \theta \cos(\gamma)
$$

(9)

and

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta
$$

(10)

2.1. Thermal Diffusivity And Effective Density

Thermal diffusivity and effective density of the nano-fluid are
\[ \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (11) \]

\[ \rho_{nf} = \varphi \rho_s + (1 - \varphi) \rho_f \quad (12) \]

### 2.2. Heat Capacity And Thermal Expansion Coefficient

Heat capacity and thermal expansion coefficient of the nano-fluid are therefore

\[ (\rho c_p)_{nf} = \varphi (\rho c_p)_s + (1 - \varphi) (\rho c_p)_f \quad (13) \]

\[ (\rho \beta)_{nf} = \varphi (\rho \beta)_s + (1 - \varphi) (\rho \beta)_f \quad (14) \]

### 2.3. Viscosity

The effective viscosity of nano-fluid was calculated by:

\[ \mu_{eff} = \mu_f \left( 1 + 2.5\varphi \left[ 1 + \eta \left( \frac{d_p}{L} \right)^{-2\varepsilon} \varphi^{\frac{2\varepsilon}{3}} (\varepsilon + 1) \right] \right) \quad (15) \]

This well-validated model is presented by Jang et al. (2007) for a fluid containing a dilute suspension of small rigid spherical particles and it accounts for the slip mechanism in nano-fluids. The empirical constants \( \varepsilon \) and \( \eta \) are 0.25 and 280 for Al\(_2\)O\(_3\), respectively. It is worth mentioning that the viscosity of the base fluid (water) is considered to vary with temperature and the flowing equation is used to evaluate the viscosity of water:

\[ \mu_{H_C} = (1.2723 \times T_w^{-5} - 8.736 \times T_w^{-4} + 33.708 \times T_w^{-3} - 246.6 \times T_w^{-2} + 518.78 \times T_w + 1153.9) \times 10^6 \]

Where, \( T_w = \log (T - 273) \).

### 2.4. Dimensionless Stagnant Thermal Conductivity:

The effective thermal conductivity of the nanoparticles in the liquid as stationary is calculated through the Hamilton and Crosser (1962) model by the formula:

\[ \frac{k_{\text{stationary}}}{k_f} = \frac{k_s + 2k_f - 2\varphi (k_f - k_s)}{k_s + 2k_f + \varphi (k_f - k_s)} \quad (17) \]

### 2.5. Total Dimensionless Thermal Conductivity Of Nano-Fluids:

\[ \frac{k_{nf}}{k_f} = \frac{k_{\text{stationary}}}{k_f} + \frac{k_c}{k_f} = \frac{k_s + 2k_f - 2\varphi (k_f - k_s)}{k_s + 2k_f + \varphi (k_f - k_s)} + \frac{k_c}{k_f} \]

\[ \frac{Nu_p d_f (2 - D_f) D_f}{\Pr (1 - D_f)^2} \left[ \frac{d_{\text{max}}}{d_{\text{min}}} \right]^{1-D_f} \left[ \frac{d_{\text{max}}}{d_{\text{min}}} \right]^{2-D_f} - 1 \]

This model was proposed by Xu et al. (2006) and it has been chosen in this study to describe the thermal conductivity of nano-fluids. \( C \) is an empirical constant (e.g. \( C = 85 \) for the deionized water and \( C = 280 \) for ethylene glycol) but independent of the type of nanoparticles. \( N_u_p \) is the Nusselt number for liquid flowing around a spherical particle and equal to two for a single particle in this work. The fluid molecular diameter \( d_f = 4.5 \times 10^{-10} \) (m) for water in present study. The fractal dimension \( D_f \) is determined by:
\[ Df = 2 - \frac{\ln \varphi}{\ln \left( \frac{d_{p, \text{min}}}{d_{p, \text{max}}} \right)} \]

Where, \(d_{p, \text{max}}\) and \(d_{p, \text{min}}\) are the maximum and minimum diameters of nanoparticles, respectively. Ratio of minimum to maximum nanoparticles \(d_{p, \text{min}}/d_{p, \text{max}}\) is \(R\).

\[ d_{p, \text{max}} = d_p \cdot \frac{D_f - 1}{D_f} \left( \frac{d_{p, \text{min}}}{d_{p, \text{max}}} \right)^{-1} \]

\[ d_{p, \text{min}} = d_p \cdot \frac{D_f - 1}{D_f} \]

NUMERICAL METHOD

Table 2: code validation, shows the comparison between the results in present study and other research.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) (Ra = 10^4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(u_{\text{max}})</td>
<td>16.052</td>
<td>16.158</td>
<td>16.1439</td>
<td>15.995</td>
</tr>
<tr>
<td>(Y)</td>
<td>0.817</td>
<td>0.819</td>
<td>0.822</td>
<td>0.814</td>
</tr>
<tr>
<td>(v_{\text{max}})</td>
<td>19.528</td>
<td>19.648</td>
<td>19.665</td>
<td>18.894</td>
</tr>
<tr>
<td>(X)</td>
<td>0.110</td>
<td>0.112</td>
<td>0.110</td>
<td>0.103</td>
</tr>
<tr>
<td>(N_{\text{ave}})</td>
<td>2.215</td>
<td>2.243</td>
<td>2.195</td>
<td>2.29</td>
</tr>
<tr>
<td>(c) (Ra = 10^5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(u_{\text{max}})</td>
<td>36.812</td>
<td>36.732</td>
<td>34.30</td>
<td>37.144</td>
</tr>
<tr>
<td>(Y)</td>
<td>0.856</td>
<td>0.858</td>
<td>0.856</td>
<td>0.855</td>
</tr>
<tr>
<td>(v_{\text{max}})</td>
<td>68.791</td>
<td>68.288</td>
<td>68.7646</td>
<td>68.91</td>
</tr>
<tr>
<td>(X)</td>
<td>0.062</td>
<td>0.063</td>
<td>0.05935</td>
<td>0.061</td>
</tr>
<tr>
<td>(N_{\text{ave}})</td>
<td>4.517</td>
<td>4.511</td>
<td>4.450</td>
<td>4.964</td>
</tr>
<tr>
<td>(d) (Ra = 10^6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(u_{\text{max}})</td>
<td>66.445</td>
<td>66.46987</td>
<td>65.5866</td>
<td>66.42</td>
</tr>
<tr>
<td>(Y)</td>
<td>0.873</td>
<td>0.86851</td>
<td>0.839</td>
<td>0.897</td>
</tr>
<tr>
<td>(v_{\text{max}})</td>
<td>221.748</td>
<td>222.33950</td>
<td>219.7361</td>
<td>226.4</td>
</tr>
<tr>
<td>(X)</td>
<td>0.0398</td>
<td>0.03804</td>
<td>0.04237</td>
<td>0.0206</td>
</tr>
<tr>
<td>(N_{\text{ave}})</td>
<td>8.795</td>
<td>8.757933</td>
<td>8.803</td>
<td>10.39</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

In this paper, mixed convection flow and heat transfer inside an inclined cavity filled with nano-fluid and having two hot square bodies is investigated. Figure 4 demonstrates variations of stream lines at different heights of hot bodies and different values of Rayleigh numbers at \(Re=50\) and for horizontal position of the cavity. Stream lines regarding the base fluid and the stream lines regarding the nano-fluid with volume fraction of 6% are denoted by solid and dashed lines, respectively. At \(Ra=10^4\), flow pattern show formation of a symmetrical clockwise primary cell which is formed due to shear force resulted from lid movement and buoyancy force resulted from temperature difference. This cell almost occupies the entire cavity. Also, two small vortices form between the two bodies and at between the body and the left lid. Increasing the height of the hot bodies does not have a significant effect on shape and size of the vortex inside the cavity. Also, presence of nanoparticles in the fluid only increases the intensity and size of the vortex, while does not make any changes to the flow pattern. Increasing Rayleigh number, strength of the primary cell increases due to increase in buoyancy force and its aid (agreement) to shear force and the entire cavity is occupied by this cell. Also, presence of nanoparticles within this range of parameters does not cause any changes to the flow pattern and only increases the cell strength.
Effect Of Rayleigh Number On Mixed Convection Flow...

Figure 4: Stream lines at different heights of hot bodies and different values of Ra numbers at $Re=50$ and $\gamma = 0^\circ$.

Figure 5 shows the isotherms line corresponding to the stream lines in figure 4. As can be seen in these diagrams, increase in Ra number causes the temperature lines near the lids to become more intense and temperature gradient to increase. Therefore, it is expected that by increasing Ra number, heat transfer also increases. On the other hand, intensity of isotherm lines near the non-adiabatic lids in base fluid is more than that of nano-fluid which is due to increase in thermal conductivity of nano-fluid compared to the base fluid. Increasing the height of the hot bodies also makes the isotherm lines near the lids less intense and causes the temperature gradient to decrease.
**CONCLUSION**
In this article, mixed convection flow and heat transfer inside an inclined cavity filled with nano-fluid with two hot square bodies was numerically simulated and the results were investigated. The following items are introduced as results of this study:

1. Adding nanoparticles to the base fluid increases the Nusselt number and heat transfer in all ranges of the parameters in this study.
2. Increase in Rayleigh number causes the heat transfer inside the cavity to increase which rate of this increase is more at higher values of volume fraction.

**APPENDIX**

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_p)</td>
<td>specific heat, (J , kg^{-1} , K^{-1})</td>
</tr>
<tr>
<td>(Gr)</td>
<td>Grashof number</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational acceleration, (m , s^{-2})</td>
</tr>
<tr>
<td>(h)</td>
<td>heat transfer coefficient, (W , m^{-2} , K^{-1})</td>
</tr>
<tr>
<td>(L)</td>
<td>enclosure length, (m)</td>
</tr>
<tr>
<td>(k)</td>
<td>thermal conductivity, (W , m^{-1} , K^{-1})</td>
</tr>
<tr>
<td>(Nu)</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>(p)</td>
<td>pressure, (N , m^{-2})</td>
</tr>
<tr>
<td>(P)</td>
<td>dimensionless pressure</td>
</tr>
</tbody>
</table>

### Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>thermal diffusivity, (m^2 , s)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>thermal expansion coefficient, (K^{-1})</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>(\mu)</td>
<td>dynamic viscosity, (Kg , m^{-1} , s^{-1})</td>
</tr>
<tr>
<td>(\nu)</td>
<td>kinematic viscosity, (m^2 , s^{-1})</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density, (kg , m^{-3})</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>volume fraction of the nanoparticles</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>cavity inclination angle</td>
</tr>
</tbody>
</table>
Effect Of Rayleigh Number On Mixed Convection Flow…

<table>
<thead>
<tr>
<th>Pr</th>
<th>Prandtl number</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>heat flux, W m⁻²</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>Ri</td>
<td>Richardson number</td>
</tr>
<tr>
<td>T</td>
<td>dimensional temperature, K</td>
</tr>
<tr>
<td>u, v</td>
<td>dimensional velocities components in x and y direction, m s⁻¹</td>
</tr>
<tr>
<td>U, V</td>
<td>dimensionless velocities components in X and Y direction</td>
</tr>
<tr>
<td>U₀</td>
<td>lid velocity</td>
</tr>
<tr>
<td>x, y</td>
<td>dimensional Cartesian coordinates, m</td>
</tr>
<tr>
<td>X, Y</td>
<td>dimensionless Cartesian coordinates</td>
</tr>
<tr>
<td>V,R</td>
<td>Velocity ratio of moving lids</td>
</tr>
</tbody>
</table>

**REFERENCES**


