The variations of heat transfer and slip velocity of FMWNT-water nano-fluid along the micro-channel in the lack and presence of a magnetic field

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HIGHLIGHTS

- The effects of Ha, Re, slip coefficient and nano-particles volume fraction were investigated.
- The stronger magnetic field leaded to less maximum of U at horizontal centerline and greater fluid velocity near the walls.
- Magnetic field affected severely the slip velocity along the walls of micro-channel.

ABSTRACT

Simulation of forced convection of FMWNT-water (functionalized multi-walled carbon nano-tubes) nano-fluid in a micro-channel under a magnetic field in slip flow regime is performed. The micro-channel wall is divided into two portions. The micro-channel entrance is insulated while the rest of length of the micro-channel has constant temperature (Tc). Moreover, the micro-channel domain is exposed to a magnetic field with constant strength of B0. High temperature nano-fluid (Th) enters the micro-channel and exposed to its cold walls. Slip velocity boundary condition along the walls of the micro-channel is considered. Governing equations are numerically solved using FORTRAN computer code based on the SIMPLE algorithm. Results are presented as the velocity, temperature, and Nusselt number profiles. Greater Reynolds number, Hartmann number, and volume fraction related to more heat transfer rate; however, the effects of Ha and $\phi$ are more noteworthy at higher Re.

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1. Introduction

In recent decades, study on different methods to enhance the heat transfer of fluids has attracted the attention of researchers. In this way, a new generation of fluids, called nano-fluids, with great potential in industrial application was considered as a result of the distribution of nanoparticles in a common liquid [1–5].

Common heat transfer fluids including water, oil and ethylene glycol generally have low thermal conductivity, while solid nanoparticles have high thermal conductivity. Hence, dispersion of nanoparticles in the base fluid can lead to increase its thermal conductivity. Therefore, nano-fluids could be considered an option for improving the heat transfer rate [6–10]. Enhancement of heat transfer coefficient of nano-fluids is subject to many factors. For example, heat transfer coefficient can be increased by changing the flow geometry, boundary conditions or enhancement of thermal conductivity of the fluid [11]. Numerous studies on heat transfer in different geometric shapes and the effects of external factors on the heat transfer have been performed. In this regard, Tahir and Mital [12] investigated the heat transfer in channels with a circular cross section. Heat transfer in channels with different geometry has been reported in the literature [13,14].

Cooling systems are the main concerns of thermal and electronic systems. In these circumstances, the use of improved and optimal cooling systems is inevitable. Tullius et al. [15] examined the effect of micro-channel geometry and fluid type on the cooling rate from micro-channels. The most common fluids used in micro-channels were air and water, which have low thermal conductivity.

Low heat transfer rate in micro-channels containing water or air led to use of various methods such as increasing the heat transfer surface, while this method increase the size of systems. Therefore, to overcome this problem, a new and effective cooling is required and nano-fluids were introduced as a new approach in this field [16].
In order to study the heat transfer in micro-channels, different models were studied [17–19]. The effects of micro-channel geometry, Reynolds number and nanoparticles volume fraction on the thermal performance of micro-channels have also been investigated. In general, it seems that using new nano-fluid shows a better performance against the heat load generated by small electronic devices [20–26].

Surface effects in micro-scale have dramatic impact than that in macro-scale. For example, no-slip condition commonly used for the macro-scale, in micro-channels is not true; consequently, slip condition should be used along the walls. Moreover, for micro and nano-scales, specific methods such as Lattice-Boltzmann method or molecular dynamics, which are based on particles, should be used [27–31]. Raisi et al. [32] investigated numerically the heat transfer in micro-channels by assuming existence or lack of slip velocity. They also reported the effect of nano-fluids on heat transfer rate in micro-channel in the presence of slip velocity. To this day, much researches assuming slip velocity have been presented in the field of heat transfer in micro-channel containing nano-fluids with different boundary conditions such as constant temperature or constant flux, which quickly slip boundary condition have also been studied [33–37].

In recent years, the flow and heat transfer in different sections of macro-scales exposed to a magnetic field have attracted the attention of the researchers [38–44]. In this case, the magnetic field leads to produce a force, called the Lorentz force, which affects the fluid flow [45–48]. Aminossadati et al. [49] studied the effects of magnetic field on a micro-channel under constant heat flux, while the slip velocity along the walls of the micro-channel was neglected.

However, a review of previous researches [53,54] showed that regarding the study of flow and heat transfer in micro-channels under the magnetic field, the slip velocity as a boundary condition has been ignored. Up to now, there is no comprehensive research on the simultaneous effects of the magnetic field, slip velocity boundary condition, forced convection on the flow and heat transfer in micro-channels. In this study, all of these conditions are considered simultaneously and efforts were also made to simulate the effects of magnetic field on the slip velocity of molecules adjacent wall. Moreover, in this study water-based nano-fluid containing functionalized multi-walled carbon nanotubes (FMWNT) is used as the working fluid.

2. Problem statement

Simulation of forced convection of FMWNT-water (functionalized multi-walled carbon nano-tubes suspended in water) nano-fluid in a two dimensional micro-channel under a magnetic field for slip flow regime is performed.

Single-walled carbon nanotubes (SWNTs) and multi-walled carbon nanotubes (MWNTs) are similar in certain respects but they also have striking differences. SWNTs structure is a cylindrical tube including six-membered carbon rings similar to graphite. They consist of a hollow cylinder of carbon ~1 nm (in present work for one cylinder) in diameter, up to 1000 times as long as it is wide. Analogously MWNTs include several tubes layers (concentric tubes) of graphene in concentric cylinders. The interlayer distance in multi-walled nanotubes is close to the distance between graphene layers in graphite, approximately 3.4 Å. The number of these concentric walls may vary from 6 to 25 or more. The diameter of MWNTs may be 30 nm (used at present article) when compared to 0.7–2.0 nm for typical SWNTs. The unique properties of carbon nanotubes enable a wide range of novel applications and improvements in the performance of existing ones. However, one can functionalize the nanotubes to enhance both the strength and dispersibility of composites.

The supposed micro-channel aspect ratio is 30; hence the fully developed condition is achieved at outlet. As shown in Fig. 1, the micro-channel wall is divided into two portions. The micro-channel entrance (X=0.3L) is insulated while the rest of length of the micro-channel has constant temperature (Tc). This configuration leads to approach the hydrodynamic fully developed condition. Moreover, the micro-channel domain is exposed to a magnetic field with constant strength of B0. Slip velocity boundary condition along the walls of the micro-channel is considered. High temperature nano-fluid (Tf) enters the micro-channel and exposed to its cold walls. Finally, it leaves the micro-channel from the other side. The empirical data for thermo-physical properties of the FMWNT-water nano-fluid are presented in Table 1. With regard to consider therevery low weight fraction of nanotubes, nano-fluid is assumed as a Newtonian fluid. Reynolds number (Re) commonly is low in a micro-channel due to express a real physical condition (Re=20 and 200). Moreover, in order to investigate the slip velocity boundary condition different values of slip coefficient such as B=0.005 and B=0.05 are assumed. Based on Table 1, three different values of weight fractions of FMWNTs are applied. Earlier investigations showed that the Hartman numbers greater than 40 have no significant effect on the flow and heat transfer; thus, a reasonable range of Hartmann number is found lower than 40. It should be noted that the nano-fluid is incompressible and homogeneous mixture.

3. Mathematical formulation

The two-dimensional Navier–Stokes equations (continuity, momentum and energy) with regard to the effect of a magnetic field strength (B0) are as follows:

\[
\frac{du}{dx} + \frac{dv}{dy} = 0 \tag{1}
\]

\[
u \left( \frac{du}{dx} + \frac{dv}{dy} \right) = \frac{1}{\rho} \frac{dp}{dx} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \partial B^2}{\rho} \tag{2}
\]

\[
u \left( \frac{du}{dx} + \frac{dv}{dy} \right) = \frac{1}{\rho} \frac{dp}{dy} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{4}
\]

\[
\sigma \text{ indicates the electrical conductivity of the nano-fluid and is equal to } 4.99 \times 10^{-2} \text{ (S/cm).}
\]
It is necessary that Eqs. (1)–(4) are written in the form of dimensionless as follows:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
\]

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \frac{Ha^2}{Re} U
\]

\[
U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = \frac{1}{Pr} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right)
\]

\[
\frac{\partial \theta}{\partial x} = \frac{\partial \theta}{\partial y} - \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
\]

In the above equations, dimensionless numbers have been defined in terms of nano-fluid properties: 

\[
Re = \frac{\mu \beta h}{\nu}, \quad Pr = \frac{\nu \alpha}{\mu}, \quad \sigma = \frac{\mu \beta h}{\rho}
\]

and \(Ha = BF \left( \frac{\sigma}{\rho} \right)^{0.5}\).

Moreover, in Eqs. (5)–(8) the following dimensionless parameters have been used:

\[
H = \frac{h}{h}, \quad L = \frac{l}{h}, \quad X = \frac{x}{h}, \quad Y = \frac{y}{h}, \quad U = \frac{U}{u_i}, \quad V = \frac{V}{u_i}, \quad P
\]

\[
\beta = \frac{p}{\rho_i u_i}, \quad \theta = \frac{T - T_i}{T_h - T_i}
\]

### Table 2

<table>
<thead>
<tr>
<th>Grid Size</th>
<th>375 x 25</th>
<th>450 x 30</th>
<th>525 x 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1.480</td>
<td>1.481</td>
<td>1.481</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.766</td>
<td>0.768</td>
<td>0.769</td>
</tr>
</tbody>
</table>

#### 4. Boundary conditions and solution algorithm

It is obvious that at the macro-scale no-slip boundary condition should be used, while in micro-channels the slip flow regime is established. The slip velocity can be calculated by the following equation:

\[
u_s = \pm \left( \frac{\partial u}{\partial y} \right)_{y=0, h}
\]

where \(\beta\) represents the slip coefficient.

Dimensionless form of Eq. (10) on the wall is expressed as follows:

\[
u_s = \pm \left( \frac{\partial u}{\partial y} \right)_{y=0, 1}
\]

in which \(B\) is called dimensionless slip coefficient.

Other dimensionless boundary conditions are as follows:

\[
U = 1, \quad V = 0, \quad \theta = 1 \quad at (X = 0, 0 \leq Y \leq 1)
\]

\[
\frac{\partial U}{\partial x} = \frac{\partial \theta}{\partial y} = 0 \quad at (X = 30, 0 \leq Y \leq 1)
\]

---

**Fig. 2.** Horizontal velocity profiles at fully developed condition from present work versus \([51]\).

**Fig. 3.** Nu\(\text{m}\) from present work versus \([52]\) (Symbols: \([52]\), Lines: Present work).

**Fig. 4.** Nu\(\text{m}\) from present work versus \([49]\) at \(\phi = 0.02\).
\[ V = 0, U = U_y, \theta = 0 \text{ at } Y = 0, 9 < X \leq 30, \] \[ \text{and} \quad (Y = 1, 9 < X \leq 30) \]

\[ V = 0, U = U_y, \frac{\partial \theta}{\partial Y} = 0 \text{ at } (Y = 0, 0 \leq X \leq 9) \] \[ \text{and} \quad (Y = 1, 0 \leq X \leq 9) \]

Based on above boundary conditions, Eqs. (5)–(8) are numerically solved using FORTRAN computer code. The finite volume method based on the SIMPLE algorithm was employed to solve the governing equations. Discretization of the nonlinear equations was performed by the power law scheme.

Local Nusselt number is:

\[
Nu_{y=0} = -\frac{k_f}{k_f} \frac{\partial \theta}{\partial y} \bigg|_{y=0}
\]

\[
Nu_{y=1} = -\frac{k_f}{k_f} \frac{\partial \theta}{\partial y} \bigg|_{y=1}
\]

Nusselt number with integration over the wall to be expressed as follows

\[
Nu_m = \frac{1}{0.7L} \int_{0}^{L} Nu_a(X) dX
\]

5. Results and discussion

Forced convective heat transfer of nano-fluid composed of water and functionalized multi-walled carbon nanotubes (FMWNT) in a two-dimensional micro-channel has been numerically investigated. The results were classified in the following subsections.

5.1. Grid independency and code verification

Table 2 presents the values of \(U\) and \(\theta\) at the central point of the micro-channel in the absence of magnetic field for various grids.

It can be found from Table 2 that difference between those of 450 \(\times\) 30 and 525 \(\times\) 35 is negligible; consequently, grid nodes of 450 \(\times\) 30 is selected for the next computations.

In order to ensure the accuracy of the results obtained by the developed FORTRAN code, results of present work versus those of Hooman and Ejlali [51] are presented in Fig. 2. In this figure, horizontal velocity profiles at fully developed condition through a micro-channel for different values of Knudsen number (Kn) are compared. Moreover, Fig. 3 shows a comparison between the averaged Nusselt number obtained by the code and that presented by Santra et al. [52] for Reynolds numbers of 100 and 200. The last selected case for confirmation would be a work of Aminossadati et al. [49] which is displayed in Fig. 4 and pertains to a forced convection flow of nanofluid in a micro-channel exposed to a
magnetic field and in the absence of slip velocity. Good agreements can be observed in Figs. 2–4.

5.2. Effects of dimensionless slip coefficient

Fig. 5 shows the dimensionless velocity profiles \( (U) \) at different sections of micro-channel for various values of \( B \). This figure shows that with increasing \( X \), velocity profile becomes smoother; as a result, the shear stress increases near the walls. It is also found from Fig. 5 that with increasing slip coefficient the slip velocity increases. For the case of \( B = 0.05 \), with increasing \( X \), the slip velocity enhances that leads to a smoother velocity profile. For example, slip velocity values are approximately 0.25, 0.40 and 0.60 at \( X = 0.1 \), 0.3 and 0.6, respectively.

5.3. Effects of Hartmann number

Fig. 6 shows \( U \) profiles along the vertical line in developed region of micro-channel for various Hartman numbers and slip coefficients of 0.005 and 0.05. We know that applying the magnetic field leads to the generation of the Lorentz force in the reverse direction of movement of nano-fluid. Hence, the stronger magnetic field can lead to less maximum of \( U \) at horizontal centerline and greater fluid velocity near to the walls. Consequently, the fully developed velocity profile would vary with \( Ha \). This behavior can be found well in Fig. 6 at \( B = 0.005 \); thus, stronger magnetic field leads to thinner boundary layer along the walls. Furthermore, the growth of thickness of the boundary layer is small at greater \( Ha \) values.

It can be also observed in Fig. 6 that the slip velocity is changed by Hartmann number at \( B = 0.05 \). As shown in this figure, magnetic field affects the slip velocity on the walls of micro-channels such that slip velocity increases with increasing Hartmann number. For example, slip velocity is equal to 0.25 for in absence of magnetic field, while it would be 0.5 and 0.65 for \( Ha = 20 \) and \( Ha = 40 \), respectively.

In order to demonstrate the effects of \( Ha \) on \( \theta \) profiles, the temperature profiles of \( \theta \) at different sections of micro-channel under various values of \( Ha \) for \( B = 0.005 \) and \( B = 0.05 \) are displayed in Figs. 7 and 8, respectively. In both figures, at \( X = 0.3L \), where the flow is not significantly affected by the cold walls, the temperature and its gradient are great. The high temperature gradient indicates that the heat transfer rate in this zone is considerable. In the middle of the micro-channel \( (0.5L \leq X \leq 0.6L) \), where nano-fluid exchanges heat with the cold walls, the slope of the temperature profile is reduced. In the area near the outlet \( (X \geq 0.7L) \), nano-fluid tends to the temperature of the walls, and the temperature gradient is reduced. Comparison of temperature profiles in the absence of magnetic field \( (Ha = 0) \) with those at \( Ha = 40 \) indicates that the nanofluid temperature in the various sections of the micro-channel decreases and tends to the wall’s one in the presence of magnetic field \( Ha = 40 \). This fact denotes more heat transfer...
existences due to more nanofluid velocity near the walls for the case of H=40.

Comparison between Figs. 7 and 8 shows that temperature profiles for the case of B=0.005 are almost similar to those for the case of B=0.05, implying the inconsiderable effects of B on temperature profiles. Fig. 9 displays the isotherms in the micro-channel at Reynolds numbers of 20 and 200. This figure denotes a good visual aspect of nanofluid flow through the micro-channel.

5.4. Effects of Carbon nano-tubes concentration

Fig. 10 depicts the of $U_i$ along the micro-channel wall at various values of $B$ for Ha=0, Ha=20 and Ha=40. $U_i$ starts from its maximum value at inlet and then it diminishes with increasing X and, eventually, becomes constant along the wall. Furthermore, slip coefficient significantly affects the slip velocity such that it increases with enhancing slip coefficient. However, this event could be severely invigorated for more Ha.

Moreover, Fig. 11 illustrates Nu$_X$ along the micro-channel wall at Re=20, B=0.005, Ha=0 and Ha=40 for different values of $\phi$. Nusselt number starts from its maximum value at $X=0.3L$ and then decreases mildly along the micro-channel length. More $\phi$ corresponds to more Nu however the effects of Ha can be ignored at low values of Reynolds like Re=20. In spite of that, the positive effect of Re on Nu can be observed well in Fig. 12. It is well known that the effects of slip coefficient are significant especially at entrance length. On the other hand, less Re leads to lower entrance length; so that the effects of slip coefficient on Nu would be negligible at Re=20 in comparison with those of Re=200.

More focus on heat transfer rate, the influences of all Ha, $\phi$, B and Re on Nu$_X$ are presented in Fig. 13. It is seen that higher values of Nu can be achieved at larger amounts of Hartman number and volume fraction at higher Reynolds number (Re=200). There is inconsistency in this figure which means the slop of line is not constant and decreases with Ha. This event obviously shows that higher Hartmann number corresponds to more Nusselt number; however this increasing trend will not achieve continuously. As a result more Ha (Ha > 50) might not be useful for this purpose.
6. Conclusion

Simulation of forced convection of FMWNT-water nano-fluid in a two dimensional micro-channel under a magnetic field for slip flow regime is performed. Slip velocity and constant wall temperature were considered as boundary conditions. The effects of Hartmann number, Reynolds number, slip coefficient and solid volume fraction on velocity, temperature and Nusselt number were investigated. The following results were obtained:

- With increasing slip coefficient the slip velocity increased.
- The stronger magnetic field led to less maximum of $U$ at horizontal centerline and greater fluid velocity near to the walls. Consequently, the fully developed velocity profile would vary with Ha. Moreover, stronger magnetic field led to thinner boundary layer along the walls. Furthermore, the growth of thickness of the boundary layer was small at greater Ha values.
- Magnetic field affected the slip velocity on the walls of micro-channels such that slip velocity increased with increasing Hartmann number.
- There was more heat transfer rate due to more nanofluid velocity near the walls in the presence of magnetic field.
- At higher Hartmann numbers, the effects of slip coefficient on the slip velocity were more than that at lower Hartmann numbers.
- More solid volume fractions corresponded to more Nu, while the effects of Ha were negligible at low values of Reynolds.
- Higher values of Nu were achieved at larger amounts of Hartman number and volume fraction at higher Reynolds number ($Re=200$).

References

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