Power Allocation for Cognitive Relay System over Rayleigh Fading Channels

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Received: Spring 2013 Accepted: Autumn 2013

Abstract
In this paper we consider two networks, a primary network and a secondary cooperative communication network. In the secondary network, a source is assisted by cognitive relay nodes which allow it to coexist with the primary network. The secondary users must work under the noise floor of the primary network to achieve low interference for the primary network. Simultaneously, the secondary users should be able to complete their duty. Therefore, the cooperative system utilizes the incremental and selective decode-and-forward (ISDF) relaying protocols. In ISDF protocol, the relay sends signal in necessary situations. Thus, the ISDF cooperative scheme is considered for the secondary network in order to reduce the interference to the primary users. Under some constraints on the transmitting power from the source and relay nodes in the secondary network, they can operate below the noise level of the primary network. Under these constraints, we minimize the symbol error rate of the secondary network. It is seen that in the ISDF protocol the interference produced with the relay to the primary users is very low.

Index Terms: Cognitive relay, Incremental Selective Decode-and-Forward, Power allocation, Symbol Error Rate.

1. Introduction
The demand for radio spectrum is expected to grow rapidly in the near future. However, radio spectrum is a limited resource and it is very busy. It seems that the allocation of this limited resource is inflexible [1]. To deal with the conflicts between spectrum scarcity and spectrum utilization, cognitive radio has been considered as an efficient approach to improve the spectrum utilization by spectrum sharing between primary and secondary networks [2,3]. The sharing could be in the form of either an opportunistic overlay of idle bands in the licensed spectrum or an underlay arrangement achieved by allowing a secondary network to coexist with the primary network in such a way that the quality of service (QoS) of the primary network is not degraded by the interference caused by the transmitting power from the secondary network [4].

A problem can arise, however, because a secondary network without diversity and controlled transmitting power has a large symbol error rate; therefore, we should use diversity, specially cooperative diversity to have better performance and higher efficiency [5-7].

Regarding cooperative diversity, one commonly used protocol for transmitting data between the source and destination through relay nodes is the decode-and-forward (DF) protocol, in which at first, the relay node decodes the received signal, then it encodes and retransmits the signal to the destination [8].

Hence, in the underlay method of cognitive radio, to overcome interference, the transmitted power of the source and relay nodes in the secondary network must be controlled, because the transmission power of the secondary users should avoid any harmful interference to active primary users. The secondary network usually uses the relay nodes which are commonly near to the primary nodes so, it is necessary to perform further control on the transmitting power of the relay nodes. In order to exploit cooperative diversity effectively and further control the relay node in wireless networks, it is possible to use incremental selective decode-and-forward protocol. The new scheme jointly combines the incremental and selective DF relaying protocols [9,10]. Incremental relaying tries to create transmission opportunity by restricting the relaying process to only the necessary conditions. If the source-destination channel's SNR is sufficiently high, the relay node does nothing. However, if SNR of the source-destination channel is highly insufficient for successful direct transmission, the relay node is called upon to perform the decode-and-forward operation on what has been received from the source [11,12].

It is noteworthy that, according to the restriction of the relay process to only the necessary conditions, the average interference caused by the relay for the primary nodes is reduced in comparison with the common relaying mode.

In [13], the system throughput of the secondary network which utilizes amplify-and-forward protocol is maximized. The power allocation is investigated under limited interference to licensed (primary) users in
cognitive radio (CR) systems. In [14], using NAF cooperative protocol, the system throughput maximization problem in the relay-assisted cognitive radio network is investigated. Optimal power allocation to the cognitive relay network in order to minimize the system outage probability also is investigated in [15]. Yue (et al.) also investigate(s) outage probability minimization for cognitive relay network with common decode-and-forward protocol [16].

In this paper, we analyze SER (Symbol Error Rate) performance of the cognitive network with the incremental and selective DF cooperative scheme over Rayleigh fading channels. The exact SER expression for M-PSK modulation is derived. In addition, optimal power allocation is investigated based on the derived approximated SER expression under the transmitting power constraint which is forced by the primary network. It should be noted that only a partial CSI (Channel State Information) is required to determine how much power should be allocated to the source and relay nodes to optimize the SER performance in the secondary network. The rest of this paper is organized as follows. Section 2 describes the system model and the relaying protocol. Also the SER performance of the system for the ISDF protocol is investigated. In Section 3, the transmission power constraints and the optimal power allocation are explained. Numerical results are presented in Section 4. Finally, conclusions are presented in Section 5.

2. System Model
Consider a secondary and a primary network in the cognitive radio architecture. As shown in Fig. 1, in the secondary network the communication is performed via a relay node in two phases. The source node broadcasts data in the first phase. In this phase, the destination and relay nodes listen to the channel and try to decode the signal. If the destination decodes the message then the destination broadcasts a bit to indicate the correctness of the decoding. Subsequently, in the second phase the source transmits another symbol. If the relay node decodes the message in the first phase, the decoded message will be sent to the destination; otherwise, the relay node remains silent and the source resends the symbol. In both phases, the primary users are communicated with them. Thus the transmitting powers of the source and relay nodes in the secondary network may cause the interference to the primary users.

Both networks are assumed to operate over independent Rayleigh fading channels. The fading coefficient for a link in secondary network, h, and the magnitude of h is Rayleigh distributed with the probability density function (PDF) given by [17]:

\[ P_h(\Omega) = \frac{2\Omega}{\pi} e^{-\frac{\Omega^2}{\pi}} \]  

(1)

where \( \Omega = E[|h|^2] \) is the average power gain of the channel. The fading coefficient from the source and relay nodes to the destination node in the primary network are denoted by \( g_s \) and \( g_r \), respectively. All of the channel coefficients \( (h_{sd}, h_{sr}, h_{rd}, h_{sd}, h_{sr}) \) are independent of each other. We also assume that all the additive white Gaussian noise (AWGN) terms in the three links \((S \rightarrow D, S \rightarrow R \text{ and } R \rightarrow D)\) have a zero mean with equal variance of \( N_0 \) (i.e., \( n \sim N(0, N_0) \)).

The source broadcasts the signal \( x \). Without loss of generality, the signal power is normalized to unity \( (E[|x|^2] = 1) \). The power transmitted by the source is assumed to be \( p_s \) and the power transmitted by the relay node is assumed to be \( p_r \). We consider the total transmitted power of the source and relay nodes to be constant. Therefore, the received signal at the destination may be written as

\[ y_{sd} = \sqrt{p_s} h_{sd}x + n_{sd} \]  

(2)

where \( h_{sd} \) is the channel gain for the source-destination link and \( n_{sd} \) is AWGN with a zero mean and variance \( N_0 \) for the source-destination link. The received signal at the relay node may be written as

\[ y_{sr} = \sqrt{p_s} h_{sr}x + n_{sr} \]  

(3)

where \( h_{sr} \) is the channel gain for the source-relay link and \( n_{sr} \) is the additive white Gaussian noise with a zero mean and variance \( N_0 \) for the source-relay link. If the destination requests the relay node to send the signal so that the second phase occurs, and the received signal in the second phase at the destination terminal may be written as

\[ y_{rd} = \sqrt{p_r} h_{rd}x + n_{rd} \]  

(4)

where \( h_{rd} \) is the channel gain for the relay-destination link and \( n_{rd} \) is the additive white Gaussian noise with a zero mean and variance \( N_0 \) for the relay-destination link. Whenever the relay and destination nodes have access to the channel state information, signal-to-noise ratios in their receivers are

\[ \gamma_{sd} = \frac{p_s |h_{sd}|^2}{N_0}, \gamma_{sr} = \frac{p_s |h_{sr}|^2}{N_0}, \gamma_{rd} = \frac{p_r |h_{rd}|^2}{N_0} \]  

(5)

where \( \gamma_{sd}, \gamma_{sr} \) and \( \gamma_{rd} \) are the respective SNRs at the source-destination, source-relay and relay-destination links.

The SER for M-PSK modulation can be written with the moment generation function (MGF). In [11], the symbol error rate is calculated for M-PSK modulation with the MGF function

\[ SER = \psi(\gamma_{sd}) \psi(\gamma_{sr}) + \psi(\gamma_{sd}) \psi(\gamma_{rd}) (1 - \psi(\gamma_{sr})) \]  

(6)

where

\[ \psi(\gamma) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \mathcal{M}_\gamma(-\frac{b}{\sin^2(\gamma)}) d\phi \]  

(7)

In Eq. (7), \( b = \sin^2(\frac{\pi}{M}) \) and \( \gamma \) is the channel’s SNR, and the moment generating function \( \mathcal{M}_\gamma \) is defined as
The MGF function for the Rayleigh distribution can be calculated as [3]:

\[ M_X(s) = \int_0^\infty p(y) e^{sy} dy \]  

where \( p(y) \) denotes the probability density function of the SNR. Therefore, the MGF function for the Rayleigh distribution can be calculated as [3]:

\[ M_X(s) = (1 + \frac{b \varphi}{\sin^2(\varphi)})^{-1} \]  

where \( \varphi \) is the average SNR, written \( \frac{P_0}{N_0} \), and \( \varphi \) is the average power gain of the channel. In practical systems, we usually have large SNR in the receivers. Assuming large SNR in the receivers allows the simplification of (9) to

\[ (1 + \frac{b \varphi}{\sin^2(\varphi)})^{-1} \approx (\frac{b \varphi}{\sin^2(\varphi)})^{-1} \]  

In decode-and-forward protocol, the relay node is usually close to the source node; therefore, the term \( 1 - \psi(y_{sr}) \) can be neglected. Consequently (6) can be presented as

\[ \text{SER} \approx \frac{1}{\pi} \sum_{m=1}^{M} (\frac{b \varphi A_{sr}}{N_0 \sin^2(\varphi)})^m d \varphi \times \frac{1}{\pi} \sum_{m=1}^{M} (\frac{b \varphi A_{rd}}{N_0 \sin^2(\varphi)})^{-m} d \varphi \]  

where with manipulation, (11) can be rewritten as

\[ \text{SER} \approx A^2 \frac{N_0^2}{b \varphi A_{sr} A_{rd}} \]  

where \( A \) is defined as follows

\[ A = \frac{1}{\pi} \int_{0}^{\pi} (\frac{b \varphi}{\sin^2(\varphi)})^m d \varphi = \frac{M-1}{2M} + \frac{\sin(\frac{2\pi}{M})}{4\pi} \]  

In the numerical result, it is shown that the approximate SER is valid for (SNR \( \geq 15 \)dB)

### 3. Power Constraints and Optimal Power Allocation

The secondary source and relay nodes should adjust their transmitted power so that the interference at the primary user node is not violated. We consider the case that, only average channel gains from cognitive nodes to the corresponding primary user nodes are available.

The power of the interference signal caused by the source in the first phase and by the relay node in the second phase to the primary receiver can be written as

\[ P_{i1} = \frac{|g_{s1}|^2}{2} \]  

and

\[ P_{i2} = \frac{|g_{s2}|^2}{2} \]  

Usually, the primary network accepts some limited level of the interference from the secondary network.

Therefore, the secondary nodes must control the transmission powers, \( P_{i1} \) and \( P_{i2} \). We have partial channel state information and the instantaneous channel state is unavailable; therefore, the interference constraints can be set in an average sense mode. The average interference must be limited to the threshold \( \Omega_{sr} \).

To prevent the source from causing severe interference to the primary users, the transmission power from the source must satisfy \( P_{i1} \leq \frac{I_{th}}{b \varphi_{sr}} \), where \( \varphi_{sr} \) is the mean channel coefficient from the source to the primary receiver.

In the ISDF protocol, the relay node receives the source's signal when needed. Hence, we define \( \xi \), the event of "signal transmitting with the relay node;" so the average interference caused by the relay node can be written as \( P_{i2} = \frac{\varphi_{sr}}{2} \Omega_{sr} \text{Pr}(\xi) \), where \( \text{Pr}(\xi) = [\psi(y_{sr})(1 - \psi(y_{sr}))], \) and \( \Omega_{sr} \) is the average channel coefficient from the relay to the primary receiver. So, the relay's transmission power must satisfy

\[ P_{i2} \leq \frac{I_{th} \theta_{sr}}{b \varphi_{sr} \text{Pr}(\xi)} \]  

In the decode-and-forward protocol, the relay node is always close to the source node; therefore, \( (1 - \psi(y_{sr})) \). Also in a high enough SNR regime, we can rewrite \( \text{Pr}(\xi) \) as \( \text{Pr}(\xi) = \frac{A_{nd}}{b \varphi_{sr} \Omega_{sr}} \) according to (7) and (8).

Thus,

\[ P_{i2} \leq \frac{I_{th} \varphi_{sr} A_{nd}}{b \varphi_{sr} \Omega_{sr}} \]  

As seen from (15), the maximum allowed transmission power of the relay node is increased when the quality of the source-destination channel is well.

The transmission powers at the source and relay are not limited only by allowed interference with the primary users, also limited with the battery capacity. Therefore, the maximum transmission powers can be rewritten as

\[ p_{1\text{max}} = \min[p_{1} \frac{I_{th}}{b \varphi_{sr}}] \]  

and

\[ p_{2\text{max}} = \min[p_{2} \frac{I_{th} \varphi_{sr} A_{nd}}{b \varphi_{sr} \Omega_{sr}}] \]  

where \( P_c \) is the limitation value forced with the battery. When the source-destination channel has a good quality, the transmission power at the relay node is limited only with \( P_c \).

Next, we discuss the power allocation problem for the ISDF protocol. If there are no interference constraints for the secondary nodes, we can solve the optimization problem as below:

\[ \begin{cases} \min_{p_{i1}, p_{i2}} \text{SER} \\ p_{i1} + p_{i2} = 2P \end{cases} \]  

Using the optimization theoretic arguments, it is not difficult to show that the optimum power allocation coefficient \( \varphi = p_{i1} P \) can be derived as [15]

\[ \beta^* = \frac{1 + \varphi_{rd} \varphi_{sr}}{4(1 - \varphi_{sr} / \varphi_{rd})}, \Omega_{sr} = \Omega_{rd} \]  

The \( \beta^* \) is derived based on the SER approximation in (9) so, it is valid for (SNR \( \geq 15 \)dB typically). Nevertheless, the approximate power allocation coefficient can also be used in a very low SNR. In this case, we may lose the performance. In the numerical result Section, it is shown that the degradation of the performance can be neglected in low SNR.

It can be seen from (19) that the optimum power coefficient is independent of the total transmission power (P) and the quality of the source-destination channels. The optimum power allocation coefficient value is restricted within the interval \( [0.5, 1] \).

When the secondary network coexists with the primary network, the power allocation coefficient in (19) is not generally valid. The total transmission power with the
secondary users should be limited. The total maximum transmission power allowed with PU is simply being
\[ P_{\text{max}} = p_{1\text{max}} + p_{2\text{max}} \] where \( p_{1\text{max}} \) and \( p_{2\text{max}} \) are derived in (16) and (17). 
\[ P_{\text{max}} = \min \left[ P_c \ln \left( \frac{P_{c1}}{N_0} \right) + \min \left[ P_c \ln \left( \frac{P_{p2\text{max}} + p_{2\text{max}}}{N_0} \right) \right] \right] \] (20)
where \( P_c \) is usually larger than the limitation imposed by the PU. Using (16) we can simplify (20) as
\[ P_{\text{max}} = \left(1 + e^{\frac{\ln P_{c1}}{N_0}} \right) \left(\frac{\ln p_{2\text{max}}}{N_0}\right) \] (21)
According to (17), we can derive the minimum transmission power allowed by the secondary source node as
\[ p_{1\text{min}} = \max[P - P_c \left(\frac{P}{1 + \left(\frac{\ln P_{c1}}{N_0}\right)^{\frac{1}{N_0}}}\right)^{\frac{1}{N_0}}] \] (22)
where \([a]^+ = \max(0, a)\). 
For \( P \leq P_{\text{max}} \), the allocated power to the source can be derived as \( p_{\text{opt}} = \beta^* P \), but the interference threshold of PU limits the power of the SUs. So, \( p_1 \) should be in the interval \( [p_{1\text{min}}, p_{1\text{max}}] \). It can be shown that the SER is a convex function in the interval \([0, P]\). Therefore, for \( \beta^* P \geq p_{1\text{max}} \) and \( \beta^* P \leq p_{1\text{min}} \), we must choose \( p_{\text{opt}} = p_{1\text{max}} \) and \( p_{\text{opt}} = p_{1\text{min}} \), respectively.
\[ p_{\text{opt}} = \max(\min(\beta^* P, p_{1\text{max}}), p_{1\text{min}}) \] (23)
For \( P \geq P_{\text{max}} \), the transmitting is not allowed because the secondary nodes cause the harmful interference to the primary network.

4. Numerical Results
This section presents evaluations conducted to verify the accuracy of the analytical results for the coexistence of the cognitive relay network that performs the incremental selective decode-and-forward protocol with a primary network. We assumed that the relay and destination nodes in the secondary network could check whether the decoding results were correct. Without loss of generality, 4-PSK modulation was considered. The noise variance is assumed to be \( N_0 = 10^{-3} \).

In Fig. (2), we investigate the SER performance based on the two power allocation coefficients; the first coefficient is the exact power allocation coefficient, which is calculated numerically and the second coefficient is the approximate power allocation coefficient. For high enough power, the SER curves with the approximate power allocation coefficient matches to the SER curves with the exact power allocation coefficient. For low transmitting power, the degradation in the SER performance can be neglected.

To evaluate the performance of the secondary network under the interference constraints, the SER of the secondary network versus \( P \) (where \( P = P_1 + P_2 \)) is plotted in Fig. (3). In this figure, the SER curves for both unconstrained (i.e., when the power allocation coefficient is set to \( \beta^* \) as in (19)) and the constrained case (i.e., when the power allocation coefficient is set to \( \beta_{\text{opt}} \) as in (23) for various \( \Omega_{g_k} \)) are drawn. As shown in this figure, when \( \Omega_{g_k} \) is increased, the maximum power, which the source and relay node can transmit is decreased, so the power allocation is not optimally set and must be set from (23) to preserve the primary interference level.

It can also be seen that for larger \( \Omega_{g_k} \), the SER curve runs away from the optimum curve (i.e., \( \beta^* \) curve). Also, the shadowed line in this figure shows the maximum total transmission power which does not cause harmful interference to the primary users. For verifying the approximation, we have also plotted the approximated SER calculated in the (9). As seen from the figure, the approximation is valid for SNR larger than 15dB (\( P \geq 15\text{dbm} \)).

Fig. (4) compares the performance of the ISDF protocol and the CDF protocol. For various \( \Omega_{g_k} \), the exact SER and the approximate SER of the both protocols versus the interference threshold \( (I_{th}) \) are plotted. Since, in the ISDF protocol the limitation on the power of the relay node is less than the CDF protocol, the ISDF gets higher performance than the CDF. When the mean channel gain from the relay node to the primary destination is large (the relay node is close to the primary node), the influence of the ISDF is dominant. The SER curve of non-cooperation secondary network is also provided as a benchmark for comparison. High enough interference threshold \( I_{th} \geq -5\text{dbm} \) results in the same approximate and exact SER.
We compare the SER performance for two protocols in Fig. (5): the first protocol is the ISDF, in which the relay node sends the signal in the necessary situations and the second protocol that in which the relay node always sends the signal in phase two. The performance of the ISDF protocol outperforms fixed DF protocol in all case. In fixed DF protocol, the total transmitting power is sorely limited by low interference threshold because to prevent harmful interference for the primary users. But in the ISDF protocol, the data relaying is restricted to the necessary situations. Whenever the relay node transmits the data, we can allocate the larger power to it because on average the interference caused by the relay node is under the interference threshold. So, the total transmitting power can be more increased.

In Fig. (6), the SER curves for various $\Omega_{gr}$ for both unconstrained and constrained cases are portrayed. As shown in figure, in constrained case, the maximum power that the source and relay node can transmit is decreased, so the power allocation is not optimally set and should be set to preserve the primary interference level. But for different $\Omega_{gr}$, the curves are not changed because in the ISDF protocol, interference threshold is not limited the relay transmitting power however, the battery capacity limit the transmitting power of the relay node to constant value.

5. Conclusions

In this paper, a model is considered that the secondary users coexist in an underlay manner with the primary users. By means of power controlling in the secondary network, the harmful interference to the primary users is controlled. For further interference reduction, we suggest an incremental selective relaying for the secondary network. With the incremental selective decode-and-forward scheme, the average interference caused by the secondary relay node is reduced in comparison with the common decode-and-forward scheme because the relay node resends data when needed. Consequently, the maximum allowed transmission power can be increased.

6. Acknowledgment

This work was partially supported by Iranian National Science Foundation (INSF) under contract No. 88114/46.

References