SER of M-PSK Modulation in Incremental-Selective
Decode-and-Forward Cooperative Communications
over Rayleigh Fading Channels

Rouhollah Aghajani Renani*, Reza Saadat*, Mohammad R. Aref** and Ghasem Mirjalili*

* Department of Electrical Engineering, Yazd University, Yazd, Iran
** Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
aghajani@stu.yazduni.ac.ir, rsaadat@yazduni.ac.ir, aref@sharif.edu, mirjalily@yazduni.ac.ir

Abstract— In this paper we consider a cooperative communication systems over Rayleigh fading channels. The system utilizes a combination of incremental and selective decode-and-forward (ISDF) relaying protocols. The symbol error rate (SER) of M-PSK modulation for the ISDF protocol is derived. Using an approximation of the SER expression, the optimal power allocation is investigated. In order to reduce the complexity of the receiver, we used a modified version of the ISDF protocol which does not use the storage or retransmission of the first phase signal. We also derived optimal power allocation coefficient for modified ISDF. It has been noted that for the optimal power allocation in the ISDF protocol, at least 2/3 of total power must be used by the direct link. Also the optimal power allocation coefficient is not dependent on SNR.

Keywords — Decode and forward; incremental relaying; selective relaying; optimal power allocation;

I. INTRODUCTION

In wireless communication systems, multipath fading causes some degradation in quality of service. To overcome this problem various researches have been performed. For instance the multi input- multi output (MIMO) technology uses spatial diversity to overcome deep fading, however, it is impractical to implement multiple antennas with low cross-correlations at each mobile set. In some scenario specifically, due to size, cost, or hardware limitations, a wireless agent may not be able to support multiple transmit antennas[1].

Cooperative among users is a practical solution to this problem, where single-antenna units cooperate to form a virtual MIMO system for achieving diversity gains through relaying channels. This form of diversity is called cooperative diversity [2-3].

The relay channel was first introduced in 1971 by Van–der Meulen as a special form of a three–terminal network [4]. The most important set of results on the capacity of the relay channel was reported by Cover and El Gamal [5-6]. Different coding schemes and a general upper bound on the capacity of this channel were presented for the first time in their paper. El Gamal and Aref established the capacity of the semi-deterministic relay channels where the received signal at the relay is a deterministic function of the transmitted signals from the sender and the relay[7-8].

Regarding cooperative diversity, one commonly used protocol for transmission between the source and destination through relays is the decode-and-forward (DF) protocol, in which the relays first decode the received signal and then encode and re transmit to the destination.

In order to effectively exploit cooperative diversity in wireless networks, Laneman et. al. proposed several relaying protocols such as selective DF and incremental DF [9].

Incremental relaying tries to save channel resources by restricting the relaying process only to the necessary conditions. This can be implemented by employing a limited feedback from the destination terminal, (e.g., a single bit indicating the success or failure of the last direct transmission attempt). If the source-destination channel's SNR is sufficiently high, the feedback indicates success of the direct transmission, so the relay does nothing. If the source-destination channel's SNR is not sufficiently high for successful direct transmission, the feedback requests the relay to perform decode-and-forward operation on what has been received from the source[9]. Performance of incremental relaying has been investigated in [10].

In the selective DF relaying scheme, the relay node can decode the received signal and then check correctness of decoding, including cyclic redundancy check (CRC) digits or signal-to-noise ratio (SNR) levels. If the symbol is correctly decoded, the relay will forward it to the destination, otherwise, the relay will remain silent. This scheme can significantly improve the performance in comparison with the fixed DF scheme, in which the relay always detects and forwards what it has received from the source[9].

To improve performance of incremental DF relaying scheme, a new relaying scheme has been proposed and its BER performance has been analyzed[11]. The new scheme jointly combines the incremental and selective DF relaying protocols, and so it is called incremental-selective DF relaying.

In this paper, we analyze SER performance of the incremental and selective DF cooperative scheme over
Rayleigh fading channels. In our model a feedback from destination indicates success of the direct transmission in contrast with the model based on the SNR threshold. The exact SER expression for M-PSK modulation is derived. In addition, optimal power allocation is investigated based on the derived asymptotic SER expressions. Note that only a partial CSI is required for determination of how much power should be allocated to the source and relay to optimize the SER performance. In addition to reduce the complexity of the receiver, we modify the ISDF protocol. In the modified ISDF protocol destination dose not storage the first phase signal and also the source does not retransmit the first phase signal in the second phase.

This paper is organized as follows. In Section II, system model and the relaying protocol are developed. SER performance is derived in Section III and it is approximated for high SNR. In Section IV, optimal power allocation is explained for the ISDF protocol. The modified ISDF protocol and its power allocation are studied in Section V. Numerical and simulation results are presented in Section VI. Finally, conclusions are given in Section VII.

II. SYSTEM MODEL

In this section we describe protocol and channel model. A wireless network with a single-user relay channel having one source, a single relay, and one destination node is considered.

A. Protocol

Fig. 1 describes the communication via a relay in two phases. The source broadcasts data in phase one. In this phase destination and relay listen to the channel. They try to decode the signal. If destination decodes the message then destination broadcasts a bit to indicate correctness of decoding. So in the second phase source transmits another symbol, otherwise, if the relay decodes the message in the first phase, the decoded message will be send to the destination, else the relay remains silent.

B. Channel Model

The relay system is assumed to operate over independent Rayleigh fading channels. Perfect synchronization and perfect channel state information are assumed to be available. The fading coefficient for a link, h and the magnitude of h is Rayleigh distributed with the probability density function (PDF) given by [12]

\[ P_h(\alpha) = \frac{2\alpha}{\bar{h}^2} e^{-\alpha^2/\bar{h}^2} \]

where (\( \bar{h} = E[|h|^2] \)) is the average power of the channel. All the channels coefficients (\( h_{sd}, h_{sr}, h_{rd} \)) are independent of each other. We also assume that all the additive white Gaussian noise (AWGN) terms in the three links (\( S \rightarrow D, S \rightarrow R \) and \( R \rightarrow D \)) are zero mean with equal variance \( N_0 \) (i.e., \( n \sim \mathcal{N}(0, N_0) \)).

C. Input–Output Relation and Signal-to-Noise Ratio

The source transmits the signal (x) with normalized power. Without loss of generality, the signal power is normalized to unity (\( E[|x|^2] = 1 \)). The transmitted power of the source is assumed to be \( p_s \) and that of the relay to be \( p_r \). Therefore the received signal at the destination may be written as

\[ y_{sd} = \sqrt{p_s} h_{sd} x + n_{sd} \]

where \( h_{sd} \) is the channel gain for the source-destination link and \( n_{sd} \) is the additive white Gaussian noise with zero mean and variance \( N_0 \) for the source-destination link. The received signal at the relay may be written as

\[ y_{sr} = \sqrt{p_s} h_{sr} x + n_{sr} \]

where \( h_{sr} \) is the channel gain for the source-relay link and \( n_{sr} \) is the additive white Gaussian noise with zero mean and variance \( N_0 \) for the source-relay link.

If the second phase happens, the received signal in the second phase at the destination terminal may be written as

\[ y_{rd} = \sqrt{p_r} h_{rd} x + n_{rd} \]

where \( h_{rd} \) is the channel gain for the relay-destination link and \( n_{rd} \) is the additive white Gaussian noise with zero mean and variance \( N_0 \) for the relay-destination link.

Whereas the relay and the destination have access to channel state information, signal to noise ratios in the first phase are

\[ y_{sd} = \frac{p_s |h_{sd}|^2}{N_0}, \quad y_{sr} = \frac{p_s |h_{sr}|^2}{N_0} \]

For DF cooperation, we assume that maximal ratio combining (MRC) is used at the destination for combining the received signals from the first and the second phase therefore signal to noise ratio at the destination may be written as

\[ y_{mrc} = \frac{p_s |h_{sd}|^2 + p_r |h_{sr}|^2}{N_0} \]

In the following section we derive SER for the system based on the channel's SNR which is derived in this section.

III. SYMBOL ERROR RATE

In this section, we analyze SER performance of incremental−selective DF over Rayleigh fading channels.

A. Exact Symbol Error Rate

In [12] SER for the direct link is calculated for M-PSK modulation using moment generating function (MGF) that is written as follows

![Figure 1. protocol of relaying data in two phase](image-url)
\[
\psi(y) = \frac{1}{\pi} \int_0^{(M-1)\pi} M_x(-\frac{b}{\sin^2(\phi)}) d\phi
\]

where \( b = \sin^2 \frac{\pi}{M} \) and \( y \) is the channel's SNR and moment generating function \( M_x \) is defined as

\[
M_x(s) = \int_0^{\infty} p(y) e^{sy} dy
\]

When SNRs in the first phase are in the form \( \frac{p_i|h_i|^2}{N_0} \) and \( h_j \)'s have Rayleigh distribution, their MGF function may be written as \[13\]

\[
M_x(\frac{-b}{\sin^2(\phi)}) = \left( 1 + \frac{b p_i e^{h_j}}{N_0 \sin^2 \phi} \right)^{-1}
\]

In the second phase due to MRC combination we have \( y_{mr}=y_{rd}+y_{sd} \) where \( y_{rd} \) is \( \frac{p_i|h_i|^2}{N_0} \), therefore its MGF function is given by\[13\]

\[
M_{y_{mr}} = M_{y_{rd}} \times M_{y_{sd}}
\]

so symbol error rate can be written as

\[
SER = \psi(y_{sd}) \psi(y_{sr}) + \psi(y_{sd}) \psi(y_{mr})(1 - \psi(y_{sr}))
\] (11)

Substituting Eq. (9) and Eq. (7) into Eq. (11) we get an integral equation that should be solved numerically.

\[
SER = \left( \frac{1}{\pi} \int_0^{(M-1)\pi} \left( 1 + \frac{b p_1 \Omega_{sd}}{N_0 \sin^2 \phi} \right)^{-1} d\phi \right) \times
\]

\[
\left( \frac{1}{\pi} \int_0^{(M-1)\pi} \left( 1 + \frac{b p_1 \Omega_{sr}}{N_0 \sin^2 \phi} \right)^{-1} d\phi \right) +
\]

\[
\left( \frac{1}{\pi} \int_0^{(M-1)\pi} \left( 1 + \frac{b p_1 \Omega_{mr}}{N_0 \sin^2 \phi} \right)^{-1} d\phi \right) \times
\]

\[
\left( \frac{1}{\pi} \int_0^{(M-1)\pi} \left( 1 + \frac{b p_2 \Omega_{sr}}{N_0 \sin^2 \phi} \right)^{-1} d\phi \right)
\]

(12)

where \( \Omega_{sr}, \Omega_{sr}, \Omega_{sr} \) are variances of the corresponding channels. To obtain a closed form equation for SER, in the next section we derive an approximation to Eq. (12).

B. Approximated Symbol Error Rate

The SER expressions involve very complex integrals that are difficult to evaluate. For large values of SNR, we can approximate Eq. (9) as \[14-15\].

\[
M_x(\frac{-b}{\sin^2(\phi)}) \approx \left( 1 + \frac{b p_i e^{h_j}}{N_0 \sin^2 \phi} \right)^{-1}
\]

therefore Eq. (7) can be rewritten as

\[
\psi(y) \equiv \frac{1}{\pi} \int_0^{(M-1)\pi} \left( \frac{b p_i e^{h_j}}{N_0 \sin^2 \phi} \right)^{-1} = \frac{A N_0}{b p_i \mu_j}
\] (14)

where \( A \) is defined as

\[
A = \frac{1}{\pi} \int_0^{(M-1)\pi} \sin^2 \phi \ d\phi = \frac{M-1}{2M} + \frac{\sin(2\pi)}{4\pi}
\]

We also define \( B \) as

\[
B = \frac{\frac{(M-1)\pi}{\sin^2 \phi}}{4\pi} + \frac{b p_i \Omega_{sr}}{N_0 \sin^2 \phi} = \frac{b p_i \Omega_{sr}}{N_0 \sin^2 \phi}
\]

therefore \( \psi(y_{mr}) \) may be written as

\[
\psi(y_{mr}) = \int_0^{(M-1)\pi} \left( \frac{b p_1 \Omega_{sr}}{N_0 \sin^2 \phi} \right)^{-1} \times \left( \frac{b p_2 \Omega_{sr}}{N_0 \sin^2 \phi} \right)^{-1} d\phi
\]

(17)

We also assume that SNR for the source relay channel is sufficiently large, therefore SER at the relay is very low, and thus the approximated overall SER may be written as

\[
SER_{app} \approx \frac{A^2 N_0^2}{b^2 p_1^2 \Omega_{sr}^2} + \frac{AB N_0^2}{b^2 p_1^2 p_2 \Omega_{sr}^2}
\]

We will show that the SER approximation is valid in high SNR regime.

IV. OPTIMAL POWER ALLOCATION

After that, we discuss the power allocation problem for the DF cooperation network. Note that only partial CSI is required for determination of how much power should be allocated to the source and the relay nodes to optimize the performance. This derivation is based on SER approximation given in Eq. (18) for M-PSK signals over Rayleigh fading channels.

Under total power constraint, i.e., \( p_1 + p_2 = P \) where \( p_1 = \beta P \) and \( p_2 = (1-\beta)P \) the approximated SER expression can be modified as

\[
SER_{app} \approx \frac{C}{\beta^2} + \frac{D}{\beta^2 (1-\beta)}
\]

where \( C = \frac{A^2 N_0^2}{b^2 p_1^2 \Omega_{sr}^2} \) and \( D = \frac{AB N_0^2}{b^2 p_1^2 p_2 \Omega_{sr}^2} \) and \( \beta \) is the power allocation coefficient. Using the optimization theoretic arguments, it is not difficult to show that the optimum power allocation coefficient can be derived as

\[
\beta = 1 + \frac{1}{4} \frac{(C)}{\sqrt{B (C)^2 + 9 (D)^2}}
\]

(20)

where

\[
\frac{D}{C} = \frac{B}{b^2 p_1^2 \Omega_{sr}^2} - \frac{D}{b^2 p_1^2 p_2 \Omega_{sr}^2}
\]

(21)

From Eq. (20), this fact can be seen that optimum power coefficient depend on SNR in an opposite manner. Where direct link's SNR is good, we interest in putting total power to direct transmission. On the other hand, when \( C \) tends to zero, \( \beta \) tends to one, i.e., destination decode the message; therefore we do not need the relay (i.e., \( p_2 = 0 \) or \( \beta = 1 \)). But if the destination could not have access to decoded message we need the relay to send a clean copy of the message to the destination. In this case we allocate some of the power to the relay.

We can see that the optimum ratio of the transmitted power \( p_1 \) at the source divided to the total power \( P \) is less than 1 and greater than 2/3, while the optimum ratio of the power \( p_2 \) used
at the relay divided to the total power $P$ is larger than 0 and less than $1/3$, i.e.,

$$\frac{2}{3} < \frac{P_1}{P} < 1, \quad 0 < \frac{P_2}{P} < \frac{1}{3}$$  \hspace{1cm} (22)

When ($\Omega_{rd}$) is very larger than $\Omega_{sd} \times \Omega_{rd}$ (i.e., quality of the source-relay channel is good), optimum power coefficient tends to $2/3$.

If the link quality between the source and the relay is bad, it is difficult for the relay to decode the transmitted symbol correctly. Thus, the forwarding role of the relay is not important and it makes sense to put all the power to the source. So when $\Omega_{sr} \ll \Omega_{rd}$, the optimum power coefficient tends to 1.

For the special scenario that the link quality between the source and the relay is the same as that of the relay and the destination (In the case $\Omega_{sr} = \Omega_{rd}$), power allocation coefficient depends on the quality of the source-destination channel. In case $\Omega_{sr} = \Omega_{rd}$, the power allocation coefficient depends on the quality of the relay-destination channel.

V. MODIFIED VERSION OF INCREMENTAL-SELECTIVE RELAYING

As seen in the previous section the optimum power allocation reversely depends on SNR. When the SNR can be changed, we consider a modified version of the incremental-selective relaying protocol which is more bandwidth efficient than the described protocol. In our modified scheme, if source signal is not correctly decoded by the destination, the destination feedbacks a bit to indicate failure in decoding. In this case, if the relay has decoded the source signal correctly, it will forward the decoded signal to the destination. Moreover, we do not assume any storage or retransmission of the first transmitted signal (to the destination). Therefore Eq. (11) can be modified as

$$\begin{align*}
SER_{mod} &= \psi(y_{sd})\psi(y_{sr}) + \\
&\quad \psi(y_{sr})(1 - \psi(y_{sr}))
\end{align*}$$  \hspace{1cm} (23)

Substituting Eq. (9) and Eq. (7) into Eq. (23) we get an integral equation that should be solved numerically.

$$
SER_{mod} = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{4} + \frac{b_1\Omega_{rd}}{N_0\sin^2\phi} \right)^{-1} d\phi \right) \times \\
\left( \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{4} + \frac{b_1\Omega_{rd}}{N_0\sin^2\phi} \right)^{-1} d\phi \right) \times \\
\left( \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{4} + \frac{b_2\Omega_{rd}}{N_0\sin^2\phi} \right)^{-1} d\phi \right) \times \\
\left( \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{4} + \frac{b_2\Omega_{rd}}{N_0\sin^2\phi} \right)^{-1} d\phi \right) \times
$$

(24)

Again note that this SER expression involves very complex integrals that are difficult to evaluate. Assuming large value of SNR, we can use approximation in Eq. (13). We also assume that when the SNR value for the source-relay channel is sufficiently high, the SER at the relay is very low, therefore, the approximated overall SER of the modified version may be written as

$$\begin{align*}
SER_{appmod} &\approx \frac{\beta^2}{b_1^2} + \frac{\alpha^2}{b_1^2} + \frac{\beta^2}{b_1^2} + \frac{\alpha^2}{b_1^2} + \frac{\beta^2}{b_1^2} + \frac{\alpha^2}{b_1^2}
\end{align*}$$  \hspace{1cm} (25)

After that, we discuss about power allocation problem for the modified incremental-selective DF cooperation network. Our derivation is based on the SER approximation given in Eq. (25) for M-PSK signals over the Rayleigh fading channels.

Under total power constraint, i.e., ($p_1 + p_2 = P$) where $p_1 = \beta P$ and $p_2 = (1 - \beta)P$ approximated SER expressions can be modified as

$$\begin{align*}
SER_{appmod} &\approx \frac{C^2}{\beta^2} + \frac{\beta^2}{\alpha^2} \\
&\approx \frac{C^2}{\beta^2} + \frac{\beta^2}{\alpha^2}
\end{align*}$$  \hspace{1cm} (26)

where $C = \frac{\alpha^2}{b_1^2}$ and $\beta = \frac{\alpha^2}{b_1^2}$ is the power allocation coefficient. Using the optimization theoretic arguments, it is not difficult to show that optimum power allocation coefficient can be derived as

$$\begin{align*}
\beta &= \frac{1 - \Omega_{rd}}{2\Omega_{sd}} + \frac{1 + \Omega_{rd}}{2\Omega_{sd}} \\
\Omega_{sr} \neq \Omega_{rd}
\end{align*}$$  \hspace{1cm} (27)

From Eq. (27) it can be seen that the optimum power coefficient does not depend on the quality of the source-destination channels. In the other hand $\beta$ can be specified independent of SNR. Optimum power allocation coefficient value is restricted to be in the interval $[0.5-1]$.

VI. EVALUATION AND SIMULATION

Computer simulations are conducted to verify accuracy of the analytical results for the incremental selective DF protocol. All the channel models are Rayleigh fading, and we assume that the relay and the destination can check whether the decoding result is correct. Without loss of generality we performed simulation for 8-PSK modulation. Power is optimally allocated to the source and the relay.

**Figure 2.** Optimum Power Allocation Coefficient in Both Protocols for Rayleigh Fading.
Fig. 2 shows the optimum power allocation coefficient in both modified an unmodified incremental-selective relaying. In modified ISDF, $\beta$ is constant with respect to SNR but in unmodified ISDF, $\beta$ has been changed with respect to SNR.

Moreover in the unmodified ISDF protocol, it can be seen that $\frac{2}{3} \leq \beta < 1$ but in the modified ISDF, power allocation coefficient is in the interval $0.5 \leq \beta < 1$. It is shown that the power allocation coefficient of the ISDF protocol is larger than the power allocation coefficient of the M-ISDF protocol.

Fig. 3 shows correct symbol rate per time slot of three protocols versus average SNR. It is clear that the average transmitted power in M-ISDF is less than that of other protocols; also the M-ISDF performance outperforms the standard DF relaying and the ISDF protocol, since the first phase signal is retransmitted (i.e., this protocol uses more bandwidth).

In Fig. 4 and Fig. 5, symbol error rate is shown for the three protocols. The first protocol is the standard DF relaying with MRC at the destination and the second protocol is incremental-selective decode-and-forward (ISDF) and the last protocol is the modified incremental-selective decode-and-forward (M-ISDF). It should be noted that we allow ISDF protocol to use more bandwidth and more average transmitted power respect to M-ISDF.

It can be seen that the ISDF protocol outperforms other protocols regarding SER performance but the ISDF protocol uses more bandwidth. M-ISDF also has better performance compared to the standard DF relaying with a low complexity receiver and less bandwidth. Moreover it can be seen that the approximated SER curves follow the exact SER curves in the high SNR regime.

VII. CONCLUSIONS

In this paper, we considered a new relaying protocol which has better SER performance compared to the standard protocol. The main objective of this work is to derive a SER expression for the Incremental selective relaying over independent Rayleigh fading channels. For ease of usage, we have also provided an approximate expression for SER.

Moreover, the optimal power allocation was investigated based on the derived approximate SER.

This protocol had complex receiver and power allocation coefficient which also was dependent on SNR, so we introduced Modified incremental selective DF protocol with some degradation of performance. However it requires a low complexity receiver and it has constant optimum power allocation coefficient with respect to SNR. Also the M-ISDF protocol is more bandwidth efficient than ISDF protocol. It should be noted that for optimal power allocation in the ISDF protocol, at least 2/3 of total power must be used by direct link while in the M-ISDF protocol, this portion is reduced to 1/2.

In the M-ISDF protocol, the power allocation coefficient does not depend on the quality of the source-destination channel. Also modulation constellation size does not affect power allocation coefficient in the M-ISDF protocol. It was shown that both ISDF and M-ISDF have better performance compared to the standard DF relaying protocol.
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