Formation Control of Mobile Robots with Obstacle Avoidance using Fuzzy Artificial Potential Field

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Abstract—In this paper, a combined formation control of nonholonomic wheeled mobile robots (WMR) is presented. In this method, the sliding mode control (SMC) strategy is used to solve the tracking problem of any robot agent. In our scenario, we assume the static and dynamic obstacles in the working environment. For guaranteeing obstacle avoidance, we use the artificial potential field method in which the coefficient of attractive and repulsive functions is determined by a Mamdani fuzzy system. Moreover, in order to maintain the desired formation considered between WMRs a fuzzy system is used to control the position and orientation of any agent with respect to the desired formation. The efficiency and simplicity of proposed control scheme has been proved by simulation on different situations.

I. INTRODUCTION

In the few recent years according to the application of multi-agent systems in the various tasks such as collaborating in exploring an environment [1], and etc., the attention of researchers have been focused on controlling such systems. In this area several control techniques including various Lyapunov based methods have been developed. In [2] a geometric framework has been used to solve the control problem of leader-follower formation. In order to control the separation and bearing of agents, the fuzzy control can be utilized to keep constant relative distance and constant angle to the leader robot [3]. Some researchers focused their study on developing backstepping controller for leader-follower based formation control of nonholonomic WMRs [4]–[6]. In [7] an artificial potential field based navigation and leader-follower formation control scheme has been proposed in which the formation control of a group of differentially driven wheeled mobile robots has been studied. The researchers of [8] suggested a new framework based on the leader-follower and cascade system by which one can simplify the communication difficulties to design controllers of a group of WMRs. The real-time navigation of WMRs was worked in [9] in which the fuzzy logic technique, wireless communication and MATLAB were used to control a WMR in an obstacle ridden environment. The proposed tracking controller represented by researchers can be categorized into four classes: linear, nonlinear, geometric and intelligent approaches [10]. In the aforementioned literature, a perfect case in which some followers follow a leader WRM with a desired formation in an environment with static and dynamic obstacles has not been studied. In this paper we considered all of these case in our research. Hence, the proposed approach presented in this paper deals with the formation control of WMRs in the presence of static and dynamic obstacles. Accordingly, each agent tracks a prespecified trajectory while it has to maintain its pose in considered formation. The trajectory tracking problem is solved by utilizing sliding mode control methodology. In addition, avoiding collisions with static and dynamic objects as well as other mobile robots by each agent must be guaranteed. In fact, obstacle avoidance has higher priority than trajectory tracking. It means that each WMR may temporarily leave the formation after passing obstacles and then back to the formation. Obstacle avoidance task is carried on using artificial potential field strategy and formation control is performed by a local fuzzy controller on each agent.

The rest of this paper is organized as follows. In section II, the leader-follower formation control using artificial potential field is presented. Then, the applied SMC approach for trajectory tracking of leader and follower WMRs are addressed in section III. Fuzzy logic leader-follower control is discussed in section IV. After that, section V addresses the application of the proposed scheme for a group of non-holonomic robots while maintaining a desired formation. Eventually, we draw some conclusions in VI.

II. LEADER FOLLOWER FORMATION CONTROL USING ARTIFICIAL POTENTIAL FIELD

A. Problem Description

Let us consider a nonholonomic mobile robot for which the aim is that it tracks a desired prespecified trajectory while guaranteeing to avoid with static and dynamic obstacles as well as other agents in the environment. Accordingly, we are given a group of WMRs modeled by the following equations

\[ \dot{x} = v \cos(\theta) \]
\[ \dot{y} = v \sin(\theta) \]
\[ \dot{\theta} = u \]

where \( x, y \in \mathbb{R} \) are the Cartesian coordinates, \( \theta \in [0, 2\pi) \) is the orientation of the robot with respect to the world frame and \( v, u \) are the linear and angular velocity inputs, respectively. One member of this group is shown in Fig.1. In addition
Avoided. We address the obstacle avoidance problem using
where $x$, $r$, $R$
and $d$
are radii of the avoidance and detection regions, respectively, as shown in Fig. 2. This function is infinite at
Fig. 2: Avoidance (radius $r$) and detection (radius $R$) regions around the robot.

\[
\begin{align*}
\frac{\partial V}{\partial x} &= \begin{cases} 
0 & \text{if } d_a < r \\
\frac{4(R^2-r^2)(d_a^2-R^2)}{(d_a^2-r^2)^3}(x-x_a) & \text{if } R > d_a > r \\
0 & \text{if } d_a \geq R 
\end{cases}
\end{align*}
\]

and
\[
\begin{align*}
\frac{\partial V}{\partial y} &= \begin{cases} 
0 & \text{if } d_a < r \\
\frac{4(R^2-r^2)(d_a^2-R^2)}{(d_a^2-r^2)^3}(y-y_a) & \text{if } R > d_a > r \\
0 & \text{if } d_a \geq R 
\end{cases}
\end{align*}
\]

Assuming the reference trajectory given by, $x_r, y_r$, with bounded derivatives, one can define
\[
\begin{align*}
E_x &= e_x + \frac{\partial x}{\partial y} \\
E_y &= e_y + \frac{\partial y}{\partial y}
\end{align*}
\]

where $e_x = x - x_r$ and $e_y = y - y_r$ are position errors in directions $x$ and $y$, respectively. Then, the desired orientation can be obtained as
\[
\theta_r = \text{Atan2}(-E_y, -E_x)
\]

In addition, the orientation error is given as $e_\theta = \theta - \theta_r$, in which $\theta_r$ is the desired direction of motion that depends on the reference trajectory, the robot’s position and the obstacle to be avoided by the robot.

Note that in order to avoid singular cases, the following conditions must be met

- The reference trajectory must be smooth and $|e_\theta| \neq \frac{\pi}{2}$
- For $r \leq da < R$ it requires that $\dot{x}_r = \dot{y}_r = 0$ causing that as the robot detects an obstacle in its path, temporarily does not consider its tracking performance. It means that collision avoidance has a higher priority than tracking.
- In order to guarantee $\dot{\theta}_r$ to be a sufficient smooth estimate of

\[
\dot{\theta}_r = \frac{E_x \dot{E}_y - \dot{E}_x E_y}{D^2},
\]

the following inequality must be satisfied for small positive values $T, \epsilon$

\[
\frac{E_x(\dot{E}_y - \dot{\theta}_r) - E_y(\dot{E}_x - \dot{\theta}_r)}{D^2} \leq \epsilon
\]

where

\[
\begin{align*}
\dot{E}_x &= \frac{E_x(t + T) - E_x(t)}{T} \\
\dot{E}_y &= \frac{E_y(t + T) - E_y(t)}{T}
\end{align*}
\]

and $D = \sqrt{E_x^2 + E_y^2}$

III. SLIDING MODE CONTROL OF LEADER AND FOLLOWER WMRs

In order to track the desired trajectory by leader and follower WMRs in a formation, the SMC approach can be employed. In this paper for $i$th follower robot, we use the following control law [14], [15]

\[
\begin{bmatrix}
v_i \\ u_i
\end{bmatrix} = \begin{bmatrix}
e_{i2}u_i + v_r \cos(e_{i3}) + \tanh(s_1) + k_2s_1 \\
u_i + \frac{u_{i2}}{1+\frac{s_1}{s_2}} + \frac{u_{i3}}{1+\frac{s_3}{s_2}}(v_r \sin(e_{i3}) + \tanh(s_2)) + k_3s_2
\end{bmatrix}
\]

where $e, k_2, k_3$ are small positive numbers and $0 < a < 1$, as well as $\alpha = \arctan(v_r e_{i2})$. In (9), the dynamics of error between actual and reference velocities $(v_i, u_r)^T$ is given by

\[
\begin{bmatrix}
\dot{e}_{i1} \\ \dot{e}_{i2} \\ \dot{e}_{i3}
\end{bmatrix} = \begin{bmatrix}
u_i e_{i2} - v_r + u_r \cos(e_{i3}) \\
u_i e_{i3} - v_r \sin(e_{i3}) \\
u_r - u_i
\end{bmatrix}
\]
In the equations (14) to (16), the distances $s_1$ and $s_2$ can be considered as

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} e_{i1} \\ e_{i3} + \arctan(v_i e_{i2}) \end{bmatrix}$$

(11)

For proving the exponentially stability of the control system, we can consider the following Lyapunov function

$$V_i = \frac{1}{2} s^T s$$

(12)

whose derivative is

$$\dot{V}_i = -k_2 e_{i1}^2 - k_2 e_{i2} e_{i2} - (e_{i3} + \arctan(v_i e_{i2}))^2 \leq 0.$$  

(13)

Therefore, the switching function can converge to zero in a finite time. When $s \to 0$, $e_{i3} \to 0$ and then the tracking posture error converges to zero.

On the other hand, the following control law can be utilized for the $j$th follower WMR [14], [15]

$$\begin{bmatrix} v_j \\ u_j \end{bmatrix} = \begin{bmatrix} v_i \cos(e_{j3}) + k_4 e_{i1} \\ u_i + (v_i + k_v)k_5 e_{j2} + (v_i + k_v)k_6 \sin(e_{j3}) \end{bmatrix} + \gamma_{uj}$$

where $k_4, k_5, k_6, k_v$ are positive real number. Note that the parameter $k_v$ ensures the asymptotic stability of the system. Other part of (14) is specified as

$$\gamma_{uj} = -u_j l_{ijd} \sin(\psi_{ijd} + e_{j3})$$

(15)

$$\gamma_{uj} = \frac{e_{j2} | (u_i (d_j + l_{ijd}) + k_6 (v_i + k_v) d_j + k_v) (1/k_5) | e_{j2} | d_j}{d_j}$$

(16)

In the equations (14) to (16), the distances $d_j, l_{ijd}$ and angle $\psi_{ijd}$ are specified according to Fig.3.

For a robot to be in its position in a desired formation, the following conditions must be satisfied:

- The angle between the robots, $\alpha$, must be equal to the heading of the leader, $\theta_L$.
- The heading of the follower, $\theta_F$, must be equal to the heading of the leader, $\theta_L$.
- The distance between the robots must be equal to a predefined value which is measurable.

Let us define angle $\phi$ as

$$\phi = (\theta_L - \alpha) - (\theta_L - \theta_F) = \theta_F - \alpha$$

(18)

When angle $\phi$ equals to zero, the robot is in the correct pose for a desired formation shape. Two inputs to the formation Fuzzy Logic Controller (FLC) are angle $\phi$ and distance $d$. Two outputs of formation FLC are linear and angular velocities $v, u$. In this FLC, we use trapezoidal membership function for inputs as shown in Figs 5 and 6. Moreover, the rules used in the FLC are as follows:

- For angular velocity output:
Fig. 4: The position leader-follower robot

Fig. 5: Angle input (degrees).

Fig. 6: Distance input (meters).

- if $\phi = 0$, Robot is in line with leader, then no action required.
- if $\phi > 0$, Robot is too far to the right of the leader, then turn left.
- if $\phi < 0$, Robot is too far to the left of the leader, then turn right.

- for translational velocity output:
  - if $d = 0$, Robot is at correct distance, then no action required.
  - if $d > 0$, Robot is too close to the leader, then slow down.
  - if $d < 0$, Robot is too far away from the leader, then speed up.

Note also that for outputs, we employ the singleton membership function.

V. SIMULATION RESULTS

A. Wheeled Mobile Robot and Moving Obstacles With Fuzzy Artificial Potential Field:

In order to verify the efficiency of proposed controller, simulation results are derived by developing algorithms in MATLAB environment.

B. Leader-Follower Nonholonomic Robot without Obstacle

We used five identical nonholonomic mobile robots is considered where leader’s trajectory is the desired formation trajectory, and simulations are carried out in MATLAB under following two scenarios. In the first scenario, we impose the following control input to leader:

\[ v_L = 1.6 \text{ m/s}, \quad u_L = -\sin(1.2t) \text{ rad/s}. \]

Moreover, the initial conditions taken into account are

\[ q_L = (5, 2, \pi/2)^T, \quad q_{F1} = (3.5, 2.2, \pi/6)^T, \quad q_{F2} = (5.5, 1.5, 2\pi/3)^T, \quad q_{F3} = (3.3, \pi/4)^T, \quad q_{F4} = (6.5, 1. \pi/6)^T. \]

We consider desired value $d_1 = 0.9 \text{ cm}$, $d_2 = 0.9 \text{ cm}$, $\phi_1 = -\pi/3 \text{ rad/s}$, and $\phi_2 = -\pi/3 \text{ rad/s}$.

For SMC, used parameters are $k_1 = k_2 = 5$, $k_v = 1$, $v_{F1} = v_{F2} = 4 \text{ m/s}$ and sample time is $dt = 0.01 \text{ s}$. Followers keep a certain distance and velocity track the leader vehicle, so we can regard it as four vehicles formation as shown in Fig. 7.

Fig. 7: Followers curvature Leader.

For second scenario, we use circular trajectory tracking for leader and three followers wheeled mobile robots with same kinematics and dynamics equations. Thus initial positions and orientations for reference, leader and follower robots given by:

\[ v_r = 0.4 * (5 - \cos(\pi * t/4)) \text{ m/s}, \quad u_r = 0.1 * (5 - \cos(pi * t/4)) \text{ rad/s}, \quad q_r = (3, 3, \pi/3)^T, \quad q_l = (2, 2, \pi/4)^T, \quad q_{F1} = (1, 2, \pi/3)^T, \quad q_{F2} = (2, 0.7, \pi/6)^T, \quad d_1 = 2 \text{ cm}, \quad d_2 = 2 \text{ cm}, \quad k_1 = 20, \quad k_2 = 10. \]

Fig. 8 shows the formation control between leader and two followers. For this scenario, Fig. 10 shows linear and angular velocity of two follower WMRs. Moreover, fig. 9 shows error dynamics for linear and angular velocities followers goes to zero. In this scenario, we use a FLC whose rules are determined by three laser sensors mounted on the left, front and right sides of the robot as shown in Fig. 11. These rules have been summarized in TABLE I.
C. Simulation Leader-Follower Nonholonomic Robot with Static and Moving Obstacle:

For simulation leader follower with static and dynamic obstacle, two scenarios are setup.

In the first scenario, two followers follow a leader in an environment with static and dynamic obstacles as shown in Fig. 12. In this scenario, initial values for parameters are as follows. The amount of radius of the avoidance and detection regions are $r = 0.3 \, \text{cm}$, $R = 1 \, \text{cm}$, respectively. Moreover the rest parameters are: $d_1 = 2.5 \, \text{cm}$, $d_2 = 2.5 \, \text{cm}$, $v_1 = 1 \, \text{m/s}$, $u_1 = 0.5 \, \text{rad/s}$, $\phi_{d1} = \pi/3 \, \text{rad/s}$, $\phi_{d2} = -\pi/3 \, \text{rad/s}$, $q_l = (2, 1, \pi/3)^T$, $q_f1 = (3, 0.5, \pi/4)^T$ and $q_{f2} = (0.3, 2.3, \pi/4)^T$.

In the second scenario, two followers seek the leader in the presence moving obstacles in a circular reference trajectory as shown in Fig. 13. Follower robots automatically make decision to turn the obstacle from left side or right side. Initial values for this scenario are:

$$r_d = 0.5 \, \text{cm}$$
$$R_D = 1 \, \text{cm}$$
$$d_1 = 1.5 \, \text{cm}$$
$$d_2 = 1.5 \, \text{cm}$$
$$\phi_{d1} = 2\pi/3 \, \text{rad/s}$$
$$\phi_{d2} = -2\pi/3 \, \text{rad/s}$$
$$q_r = (3, 3, \pi/3)^T$$
$$q_l = (2, 2, \pi/4)^T$$
$$q_{f1} = (0.5, 2, \pi/3)^T$$
$$q_{f2} = (2, 0.7, \pi/6)^T$$

VI. CONCLUSION

In this paper, we have studied the problem of trajectory tracking and collision avoidance for non-holonomic systems. Accordingly, we developed a novel leader-follower formation.
control in which a group of follower WMRs seek a leader in a desired formation shape so that the leader tracks a reference trajectory and follower WMRs follow their leader in an environment with static and dynamic obstacles. The sliding mode control was used to trajectory tracking of agents and artificial potential field was responsible for avoiding the agents with obstacles as well as other WMRs. For keeping the formation shape while moving the robots, a fuzzy logic controller was employed. The simulation was conducted based on Matlab/Simulink. Simulation results in different situations indicate the proposed framework can work well.

REFERENCES


