

## CHAPTER 5

**5.1** (a)  $\frac{120f}{P} \approx 1746$

$$P = \frac{120 \times 60}{1746} \rightarrow 4$$

(b)  $n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$

$$S = \frac{1800 - 1746}{1800} = 0.03$$

(c)  $f_2 = 0.03 \times 60 = 1.8 \text{ Hz}$

(d) (i) 1800 rpm  
(ii) 0 rpm

**5.2** (a)  $0.03 = \frac{1200 - n}{1200}$ ,  $n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm}$ .

$n = 1164 \text{ rpm} \rightarrow$  direction same as the rotating field.

(b)  $f_2 = 0.03 \times 60 = 1.8 \text{ Hz}$

(c) 1200 rpm

(d) 1200 rpm

(e) (i)  $n_M = 0.03 \times 1200 = 36 \text{ rpm} \rightarrow$  direction same as the rotor motion.

(ii) 1200 rpm

(iii) 0 rpm

**5.3** (a)  $-0.03 = \frac{1200 - n}{1200}$

$n = 1236 \text{ rpm} \rightarrow$  direction same as the rotating field.

(b)  $f_2 = 0.03 \times 60 = 1.8 \text{ Hz}$ .

(c) 1200 rpm.

(d) 1200 rpm.

(e) (i)  $n_M = -0.03 \times 1200 = -36 \text{ rpm} \rightarrow$  direction opposite to rotor motion.

(ii) 1200 rpm.

(iii) 0 rpm

**5.4** (a)  $n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm.}$

$$s = 1.5 = \frac{1200 - n}{1200}$$

$n = -600 \text{ rpm} \rightarrow$  direction opposite to the rotating field.

(b)  $f_2 = 1.5 \times 60 = 90 \text{ Hz.}$

(c)  $1200 \text{ rpm}$

(d)  $1200 \text{ rpm.}$

(e) (i)  $n_{rel} = 1.5 \times 1200 = 1800 \text{ rpm}$   
 $\rightarrow$  direction opposite to rotor motion  
 (or same as the rotating field).

(ii)  $1200 \text{ rpm}$

(iii)  $0 \text{ rpm.}$

**5.5** (a)(i)  $n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm}$

$$s = \frac{1200 - 1140}{1200} = 0.05$$

(ii)  $E_{2s} = s E_2 = s \frac{E_1}{a}$

$$a = \frac{1}{0.5} = 2.0$$

$$E_{2s} = 0.05 \times \frac{1}{2.0} \times \frac{208}{\sqrt{3}} = 3 \text{ V}$$

$$f_2 = 0.05 \times 60 = 3 \text{ Hz}$$

(iii) - slip rpm  $n_2 = s n_s = 0.05 \times 1200 = 60 \text{ rpm}$

- with respect to stator  $\rightarrow 1200 \text{ rpm.}$

(b) Inverted induction motor

(i) opposite

(ii)  $E_{1s} = s E_1 = s a E_2 = 0$

$$s = \frac{1200 - 1164}{1200} = 0.03$$

$$E_{1s} = 0.03 \times 2 \times \frac{208}{\sqrt{3}} = 7.2 \text{ V}$$

$$f_1 = 0.03 \times 60 = 1.8 \text{ Hz}$$

5.6(a) No load test

$$Z_{NL} = \frac{460/\sqrt{3}}{40} = 6.64 \Omega$$

$$R_{NL} = \frac{P_{NL}}{3 I_1^2} = \frac{4200}{3 \times 40^2} = 0.875 \Omega$$

$$X_{NL} = (6.64^2 - 0.875^2)^{1/2} = 6.58 \Omega$$

$$x_1 + x_m = 6.58 \Omega$$

Blocked-rotor test

$$R_{BL} = \frac{8000}{3 \times 140^2} = 0.136 \Omega$$

$$R_1 = \frac{0.152}{2} \quad (\text{from resistance between two stator terminals})$$
$$= 0.076 \Omega$$

$$R_2' = 0.136 - 0.076 = 0.06 \Omega$$

$$Z_{BL} = \frac{100/\sqrt{3}}{140} = 0.412 \Omega$$

$$X_{BL} = (0.412^2 - 0.136^2)^{1/2} = 0.389 \Omega$$

$$x_1 + x_2' = 0.389 \Omega$$

$$x_1 = x_2' = 0.1945 \Omega$$

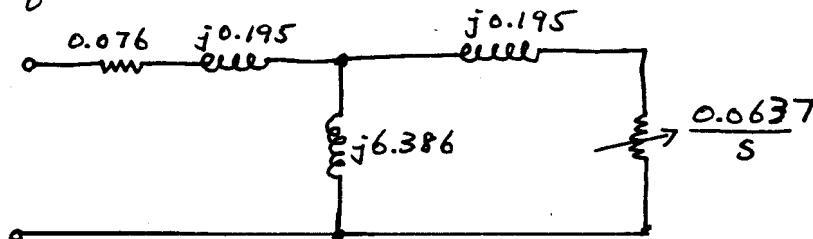
$$x_m = 6.58 - 0.1945 = 6.3855 \Omega$$

Precise value for  $R_2'$

$$R = R_{BL} - R_1 = 0.136 - 0.076 = 0.06 \Omega$$

$$R_2' = \left( \frac{0.1945 + 6.3855}{6.3855} \right)^2 \times 0.06 = 0.0637 \Omega$$

Equivalent circuit is



$$\boxed{5.6} \quad (b) \quad n_s = \frac{120f}{P} = \frac{120 \times 60}{8} = 900 \text{ rpm}$$

$$s = \frac{900 - 873}{900} = 0.03$$

$$\frac{R_2'}{s} = \frac{0.0637}{0.03} = 2.123 \Omega$$

Input Impedance  $Z_1$

$$Z_1 = 0.076 + j0.195 + \frac{(j6.386)(2.123 + j0.195)}{2.123 + j(6.386 + 0.195)} = 2.121 \angle 27.16^\circ$$

$$\text{Input Current } I_1 = \frac{460/\sqrt{3}}{2.121 \angle 27.16^\circ} = 125.22 \angle -27.16^\circ$$

$$\text{Input Power } P_{in} = 3 \times \frac{460}{\sqrt{3}} \times 125.22 \cos 27.16^\circ = 88.767 \text{ kW}$$

$$\text{Stator cu-loss, } P_{st} = 3 \times 125.22^2 \times 0.076 = 3.575 \text{ kW}$$

$$\text{Airgap Power } P_{ag} = 88.767 - 3.575 = 85.192 \text{ kW}$$

$$\text{Rotor - Cu loss} = s P_{ag} = 0.03 \times 85.192 = 2.556 \text{ kW}$$

Mechanical Power developed

$$P_{mech} = (1-s) P_{ag} = 0.97 \times 85.192 = 82.636 \text{ kW}$$

$$P_{out} = P_{mech} - \text{Protational loss}$$

From no-load test

$$P_{rot} = 4200 - 3 \times 0.076 \times 40^2 = 3835.2 \text{ W}$$

$$P_{out} = 82.636 \times 10^3 - 3835.2 = 78.80 \text{ kW}$$

$$\text{Eff} = \frac{P_{out}}{P_{in}} \times 100\% = \frac{78.80}{88.767} \times 100 = 88.77\%$$

5.7 (a)  $P_{\text{rot}} = 500 - 3 \times 6.5^2 \times \frac{0.54}{2} = 465.78 \text{ W}$

(b) From no-load test  $R_1 = \frac{0.54}{2} = 0.27 \Omega$

$V_1 = \frac{208}{\sqrt{3}} = 120.1 \text{ V}$

$Z_{NL} = \frac{120.1}{6.5} = 18.48 \Omega$

$R_{NL} = \frac{500}{3 \times 6.5^2} = 3.94 \Omega$

$X_{NL} = \sqrt{18.48^2 - 3.94^2} = 18.05 \Omega$

$X_1 + X_m = X_{NL} = 18.05 \Omega$

From blocked-rotor test

$R_{BL} = \frac{1250}{3 \times 25^2} = 0.6667 \Omega$

$R_2' = 0.6667 - 0.27 = 0.3967 \Omega$

$Z_{BL} = \frac{44/\sqrt{3}}{25} = 1.0162 \Omega$

$X_{BL} = \sqrt{1.0162^2 - 0.6667^2} = 0.7669 \Omega$

$X_1 = X_2' = 0.3834 \Omega$

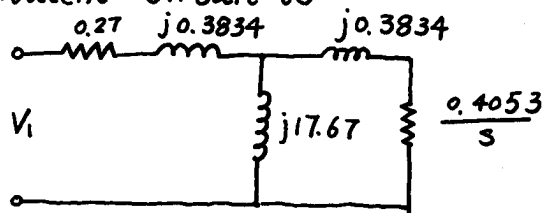
$X_m = 18.05 - 0.3834 = 17.6666 \Omega$

Precise value for  $R_2'$

$R = R_{BL} - R_1 = 0.6667 - 0.27 = 0.3967 \Omega$

$R_2' = \frac{0.3834 + 17.6667}{17.6667} \times 0.3967 = 0.4053 \Omega$

Equivalent Circuit is



(c) Neglecting  $X_m$ ,  $S_b \approx \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.4053}{\sqrt{0.27^2 + 0.7668^2}} = 0.4985$

Maximum torque occurs at 49.85% of synchronous speed.

It is class-D type motor

(d)  $\frac{R_2'}{s} = \frac{0.4053}{0.1} = 4.053 \Omega$

$Z_1 = 0.27 + j0.3834 + \frac{j17.67(4.053 + j0.3834)}{4.053 + j18.05} = 4.273 \angle 21.83^\circ$

5.7 continued:

$$I_1 = \frac{208/\sqrt{3}}{4.273 \angle 21.83^\circ} = 28.104 \angle -21.83^\circ$$

$$P_s = 3 \times 120.1 \times 28.104 \times \cos(-21.83^\circ) = 9396.809 \text{ W}$$

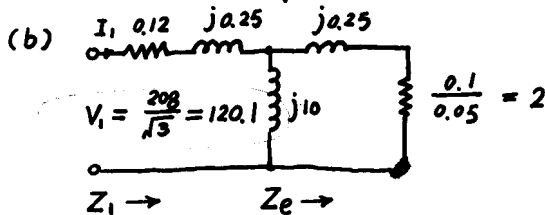
$$P_{ag} = 9396.809 - 3 \times 28.104^2 \times 0.27 = 8757.045 \text{ W}$$

$$P_{mech} = (1 - 0.1) \times 8757.045 = 7881.341 \text{ W}$$

$$P_{out} = P_{shaft} = P_{mech} - P_{rot} = 7881.341 - 465.78 = 7415.561 \text{ W}$$

$$HP = \frac{7415.561}{746} = 9.94 \text{ (HP)}$$

5.8 (a)  $n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$   $\omega_s = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec}$



$$Z_1 = 0.12 + j0.25 + R_e + jX_e = 0.12 + j0.25 + \frac{j10 \times (2 + j0.25)}{2 + j10.25}$$

$$= 0.12 + j0.25 + 1.9538 + j0.8517 = 2.1314 \angle 23.5533^\circ \Omega$$

$$R_e = 1.9538, X_e = 0.8517$$

$$I_1 = \frac{120.1}{2.1314 \angle 23.5533^\circ} = 56.3479 \angle -23.5533^\circ$$

$$(c) P_1 = 3 \times (56.3479)^2 \times 0.12 = 1143.0309 \text{ W}$$

$$(d) P_s = 3 \times 120.1 \times 56.3479 \times \cos(-23.5533^\circ) = 18610.9794 \text{ W}$$

$$P_{ag} = P_s - P_1 = 17467.9485 \text{ W}$$

$$(e) P_2 = s P_{ag} = 0.05 \times 17467.9485 = 873.3974 \text{ W}$$

$$(f) P_{mech} = (1 - s) P_{ag} = 16594.5511 \text{ W}$$

$$P_{shaft} = P_{mech} - P_{rotational} = 16194.5511 \text{ W}$$

$$(g) T = \frac{P_{ag}}{188.5} = \frac{17467.9485}{188.5} = 92.6682 \text{ N.m}$$

$$T_{shaft} = \frac{P_{shaft}}{188.5} = \frac{16194.5511}{188.5} = 85.9127 \text{ N.m}$$

$$(h) \text{Eff} = \frac{P_{shaft}}{P_s} = 0.8702 \times 100\% = 87.02\%$$

5.9 (a) To operate the motor at the same flux level, in order to avoid saturation, the supply voltage should be proportional to frequency

$$V = \frac{50}{60} \times 460 = 383.33 \text{ V (Line)}$$

5.9 Continued

$$(b) (i) n_s = \frac{120 \times 50}{4} = 1500 \text{ rpm} \quad n = 1500(1 - 0.03) = 1455 \text{ rpm}$$

$$f_2 = sf_1 = 0.03 \times 50 = 1.5 \text{ Hz}$$

(ii) In the equivalent circuit, resistances will remain same but the reactance will be proportional to frequency

$$R_1 = 0.25 \Omega, R_2' = 0.2 \Omega, X_1 = X_2' = 0.5 \times \frac{50}{60} = 0.42 \Omega$$

$$X_m = 30 \times \frac{50}{60} = 25 \Omega$$

$$\frac{R_2'}{s} = \frac{0.2}{0.03} = 6.67 \Omega$$

$$Z_1 = 0.2 + j0.42 + \frac{(j25) \times (6.67 + j0.42)}{6.67 + j25.42} = 6.687 \angle 21.181^\circ$$

$$V_1 = \frac{383.33}{\sqrt{3}} = 221.32 \quad I_1 = \frac{221.32}{6.687 \angle 21.181^\circ} = 33.097 \angle -21.181^\circ$$

$$\text{PF} = \cos(-21.181^\circ) = 0.9324$$

$$I_2 = \frac{j25}{6.67 + j25.42} \times 33.097 \angle -21.181^\circ = 31.475 \angle -6.479^\circ$$

$$\omega_s = \frac{1500}{60} \times 2\pi = 157.1 \text{ radian/Sec.}$$

$$P_{ag} = 3 \times 31.475^2 \times 6.67 = 19823.427 \text{ W}$$

$$T = \frac{19823.427}{157.1} = 126.184 \text{ N.m.}$$

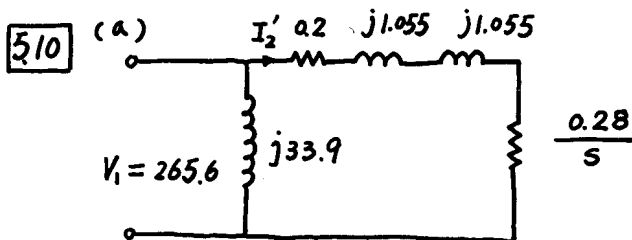
$$P_{mech} = (1 - 0.03) \times 19823.427 = 19228.724 \text{ W}$$

$$P_{rot} = 1700 \times \frac{1455}{1740} = 1421.552 \text{ W}$$

$$P_{out} = P_{mech} - P_{rot} = 17807.172 \text{ W}$$

$$P_{in} = 3 \times 221.32 \times 33.097 \times 0.9324 = 20489.568 \text{ W}$$

$$\text{Eff} = \frac{17807.172}{20489.568} = 86.9\%$$



$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \quad \omega_{syn} = \frac{1200}{60} \times 2\pi = 125.66 \text{ rad/Sec}$$

$$T_{st} = \frac{3}{125.66} \times \frac{265.6^2}{(0.48)^2 + (2.11)^2} \times 0.28 = 100.7 \text{ N.m.}$$

5.10 continued:

$$(b) \frac{0.28}{S_b} = \sqrt{0.2^2 + (2.11)^2} = 2.12$$

$$S_b = 0.132$$

$$T_b = \frac{3 \times 265.6^2}{125.66 \times (2.12/2^2 + 2.11^2)} \times 2.12/2 = 399.1 \text{ N.m.}$$

$$(c) T = \frac{1}{\omega_{syn}} \times \frac{V_1^2}{(R_1 + \frac{R_2'}{S})^2 + (X_1 + X_2')^2} \times \frac{R_2'}{S}$$

Near synchronous speed i.e.  $s$  very small,

$$R_1 + \frac{R_2'}{S} \gg X_1 + X_2' \text{ and } \frac{R_2'}{S} \gg R_1$$

$$\text{Hence } T = \frac{3}{\omega_{syn}} \times \frac{V_1^2}{R_2'} \times S = \frac{3}{125.66} \times \frac{265.6^2}{0.28} \times S = 6014.8S$$

$$6014.8S = 1.8 \omega_m = 1.8(1-S)\omega_{syn} = 1.8 \times (1-S) \times 125.66$$

$$S = \frac{226.188}{6240.988} = 0.0362 \quad n = (1 - 0.0362) \times 1200 = 1156 \text{ rpm}$$

$$\boxed{5.11} \quad V_1 = \frac{208}{\sqrt{3}} = 120.1 \text{ V} \quad X_1 = X_2' = 377 \times 0.25 \times 10^{-3} = 0.09425 \Omega$$

$$X_m = 377 \times 15 \times 10^{-3} = 5.655 \Omega$$

$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \quad \omega_s = \frac{1200}{60} \times 2\pi = 125.664 \text{ rad/sec}$$

From equation 5.55 with  $V_{th}$  replaced by  $V_1$ ,

$$T_m \simeq \frac{3}{125.664} \times \frac{120.1^2}{0.11} \times S = 3130S$$

$$T_L = 12.7 \times 10^{-3} \times (125.664(1-S))^2 = 200.55(1-S)^2$$

$$T_m = T_L$$

$$3130S = 200.55(1-S)^2 \quad S = 0.05697, 0.17553$$

$$n = 1200(1 - 0.05697) = 1131.6 \text{ rpm} \rightarrow 118.5 \text{ rad/sec}$$

$$T = 3130 \times 0.05697 = 178.32 \text{ N.m.} \quad P = 178.32 \times \frac{1131.6}{60} \times 2\pi = 21.131 \text{ kW}$$

$$\boxed{5.14} \quad (a) P = 4$$

$$(b) I_2' = \frac{265.59}{0.07 + 0.05 + j(1.2 + 1.1)} = \frac{265.59}{0.12 + j2.3} = 115.3235 \angle -89.7^\circ \text{ A}$$

$$T_{st} = \frac{3}{188.496} \times (115.3235)^2 \times 0.2 = 42.33 \text{ N.m.}$$

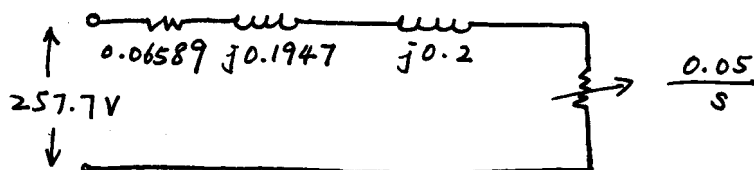
$$(c) S_{T_{max}} = 1 = \frac{0.2 + R_{ext}'}{\sqrt{(0.25)^2 + (2.3)^2}} = \frac{0.2 + R_{ext}'}{2.3135}$$

$$\rightarrow R_{ext}' = 2.1135 = R_{ext}$$

5.12(a) Thevenin Eq. cct.

$$V_{th} = \frac{x_m}{x_1 + x_m} V_1 = \frac{6.5}{0.2 + 6.5} \times 265.6 = 257.7$$

$$R_{th} + jX_{th} = \frac{(j6.5)(j0.2 + 0.07)}{0.07 + j0.2 + j6.5} = 0.06589 + j0.1947$$



$$(b) \quad T_{st} = \frac{3 \times 257.7^2 \times 0.05}{94.25 [(0.06589 + 0.05)^2 + (0.19467 + 0.2)^2]}$$

$$= 624.7 \text{ N.m}$$

$$T_{max} = \frac{3 \times 257.7^2}{2 \times 94.25 [0.06589 + \sqrt{0.06589^2 + (0.1947 + 0.2)^2}]}$$

$$= 2267.8 \text{ N.m}$$

$$s_{Tmax} = \frac{0.05}{\sqrt{0.06589^2 + (0.1947 + 0.2)^2}} = 0.1249$$

speed in rpm for which max torque occurs

$$= (1 - s_{Tmax}) n_s = (1 - 0.1249) 900 = 787.5 \text{ rpm}$$

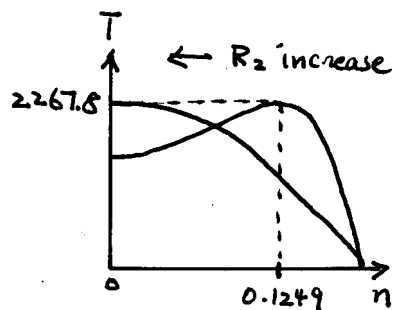
$$(c) \quad s_{Tmax} = \frac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2)^2}} \propto r_2'$$

$$s_{Tmax} = k r_2'$$

$$\frac{s_{Tmax}}{r_2'} = \frac{s / \text{start}}{r_2' \text{ at start}}$$

$$\therefore r_2' / \text{start} = \frac{s / \text{start}}{s_{Tmax}} \times r_2' = \frac{1 \times 0.05}{0.1249} = 0.4 \Omega$$

$$\therefore r_2 / \text{ext} = (0.4 - 0.05) / 1.2^2 = 0.243 \Omega$$



5.13 (a)  $\eta = 1760 \approx \frac{120 \times 60}{P}$

$P = 4.09 \rightarrow 4$

(b)  $V_1 = \frac{460}{\sqrt{3}} = 265.59$

$V_{th} = \frac{35}{1.2 + 35} V_1$

$= 0.967 \times 265.59 = 256.79 \text{ V} \Rightarrow$

$R_{th} \approx 0.967^2 \times 0.25 = 0.234 \Omega$

$X_{th} = 1.2 \Omega = X_1$

$\omega_{syn} = \frac{1800}{60} \times 2\pi = 188.496 \text{ rad./sec.}$

$T_{st} = \frac{3}{188.496} \frac{256.79^2}{(0.234 + 0.2)^2 + (1.2 + 1.1)^2} \times 0.2$   
 $= 38.31 \text{ N.m}$

(c)  $S_{Tmax} = 1 = \frac{R_2' + R_{ext}'}{\sqrt{0.234^2 + (1.2 + 1.1)^2}} = \frac{R_2' + R_{ext}'}{2.31}$

$R_{ext}' = 2.31 - 0.2 = 2.11 \Omega = R_{ext.}$

5.14 See on page 68.

5.15  $s = \frac{1800 - 1746}{1800} = 0.03$

$P_{shaft} = 100 \times 746 = 74600 \text{ W}$

$P_{mech} = 74600 + 3500 = 78100 \text{ W}$

$P_{ag} = \frac{78100}{1 - 0.03} = 80515.5$

$P_{in} = 80515.5 + 3000 = 83515.5 \text{ W}$

$\eta = \frac{74600}{83515.5} \times 100 = 89.3 \%$

5.16 (a)  $P_{ag} = 40 - 0.5 = 39.5 \text{ kW}$

$P_{cu2} = (0.025)(39.5) = 0.9875 \text{ kW}$

(b)  $P_{mech} = (1 - 0.025)(39.5) = 38.513 \text{ kW}$

