B08

Depth Estimation of Cavities from Microgravity Data through Multi Adaptive Neuro Fuzzy Interference System

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SUMMARY

We aim to estimate the depth of subsurface cavities from gravity data by a new method through a Multiple Adaptive Neuro Fuzzy Interference System (MANFIS); this method is an intelligent way to interpret microgravity data and gain an estimation of depth and shape of the most probable cavities. The MANFIS model was trained for two main models of cavities: sphere and cylinder in the related domains of radius and depth. We tested different MANFIS’s with different number of rules and obtained the optimum value for number of in the hidden layer. Then it was tested in the presence of 20% Gaussian noise and showed good robustness to noise. The method was also tested for real microgravity data from Bahamas Free Port. The results are in good agreement with ground-truthed drilled values for the depth of subsurface cavities.
Introduction

Subsurface cavities have significant negative density contrasts and are observed in residual gravity data as sharp negative anomalies. Interpretation methods such as the Analytical signal, Euler deconvolution (Thompson 1982), least-squares minimization (Abdelrahman et al. 2001), Fourier transform (Sharma and Geldart 1968) have been employed but each method has its own disadvantages. For example, the analytical signal method can only determine the location of the edges of object and the Euler method produces different responses for window size or structural index and depends crucially on the experience of the interpreter. A new intelligent method which is not dependent on the experience of the interpreter is the Multiple Adaptive Neuro Fuzzy Interference System (MANFIS) model which can estimate the depth of cavities in a desired training domain. The complicated calculations for new gravity data need not be repeated after initial training. The method is resilient in the presence of noise and different depth domains and has been tested on noisy real data.

Multiple Adaptive Neuro Fuzzy Interference System (MANFIS)

MANFIS derives its name from adaptive neuro fuzzy inference system. MANFIS is an extension of the neuro-fuzzy system ANFIS, (Jang 1993) to produce multiple outputs. A neuro-fuzzy system can serve as a nonparametric regression tool, which models the regression relationship without reference to any pre-specified functional form. Originally, ANFIS could only produce a single output but MANFIS aggregates many independent ANFIS to obtain multiple outputs. The architecture of MANFIS is depicted in Figure 1.

Figure 1 Architecture of MANFIS. Figure 2 An ANFIS architecture for a two rule Sugeno system.

Adaptive Neuro Fuzzy Inference Systems (ANFIS)

ANFIS approximates the functional relations between responses and input variables of the process under study by gradually fine-tuning the values of parameters. The ANFIS architecture is shown in Figure 2. The circular nodes are fixed whereas the square nodes have parameters to be learnt.

A Two Rule Sugeno ANFIS has rules of the form:

\[
\text{If } x \text{ is } A_i \text{ and } y \text{ is } B_i \text{ THEN } f_i = p_i x + q_i y + r_i
\]

For training the network, there is a forward pass which propagates the input vector through the network layer by layer and in the backward pass, the error is sent back through the network in a similar manner to back propagation.

Layer 1: The output of each node is:

\[
O_{i,j} = \mu_A(x) \quad \text{for } i = 1,2, \quad O_{j} = \mu_{B_{k,j}}(y) \quad \text{for } i = 3,4
\]

So, \(O_{i,j}(x)\) is essentially the membership grade for \(x\) and \(y\). The membership functions can be anything but we used the bell shaped function given by:

\[
\mu_A(x) = \frac{1}{1 + \left(\frac{x-a_i}{a_i}\right)^n}
\]
Where $a_i, b_i, c_i$ are parameters to be learnt. These are the premise parameters.

Layer 2: Every node in this layer is fixed. This is where the t-norm is used to ‘AND’ the membership grades - for example the product:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \quad i = 1,2$$

(4)

Layer 3: Layer 3 contains fixed nodes which calculate the ratio of the firing strengths of the rules:

$$O_{3,i} = \frac{w_i}{w_i + w_j}$$

(5)

Layer 4: The nodes in this layer are adaptive and perform the consequences of the rules:

$$O_{4,i} = w_i f_i = \frac{w_i (p_i x + q_i y + r_i)}$$

(6)

The parameters to be determined in this layer ($p_i, q_i, r_i$) are referred to as the consequent parameters.

Layer 5: There is a single node here that computes the overall output:

$$O_{5,i} = \sum_i w_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$

(7)

This then is typically how the input vector is fed through the network layer by layer. There are several approaches for training but we use the hybrid learning algorithm proposed by Jang, Sun and Mizutani (1997) a combination of Steepest Descent and Least Squares Estimation (LSE).

### Training strategies and MANFIS network architecture

To design and train the MANFIS model requires some available data as a set of input-outputs. With gravity data, if all the measured points are applied as inputs the training becomes very time-consuming. To prevent this we select some features from the gravity data by preparing training data for two models: spheres and cylinders. The features are calculated using equation 8 (Gret et al. 2000).

$$F_1 = \int_{x_0}^{x_5} g(x)dx, F_2 = xg_{50}, F_3 = xg_{75}$$

(8)

$F_1,F_2,F_3$: Features calculated from gravity data, $g(x)$: gravity value at $x$, $xg_{50}$: the point where the gravity value is 50% of the maximum; $xg_{75}$: is 75% of the maximum amplitude. The domain of the integral in equation 8 is from the first gravity point to the last point of profile ($x_5$).

### Figure 3 Shematic of the MANFIS with inputs and outputs.

In Fig.3 the inputs of the Model (MANFIS) are $F_1, F_2, F_3$ and outputs are $R, Z$ where $R$ is the radius of the cavity and $Z$ is the depth of the cavity. So the training set is $\{F_1, F_2, F_3\}$, $\{R, Z\}$. To prepare gravity training data we use equations 9 (Abdelrahman et al. 2001):

$$g(x,z) = \frac{AZ}{(x^2 + z^2)^{1/2}}$$

$$A = \begin{cases} \frac{4}{3} \pi GPR^3 & : \text{sphere} \\ \frac{2}{3} \pi GPR^2 & : \text{Horizontal Cylinder} \end{cases}$$

(9)

$\{R: \text{Radius}, Z: \text{Depth}, X: \text{Horizontal Distance}, G: \text{universal gravity constant} , P: \text{Contrast density}, q: \text{shape factor for sphere } q=1.5, \text{for cylinder}=1\}$. 

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F1,F2,F3 are calculated for values of depth and radius in the domain of \([R_{\text{min}}, R_{\text{max}}], [Z_{\text{min}}, Z_{\text{max}}]\). So the MANFIS network can detect cavities which have radius between \(R_{\text{min}}, R_{\text{max}}\) and depth between \(Z_{\text{min}}, Z_{\text{max}}\). For modelling purposes the gravity data are normalized between zero and one. The data is divided into 2 parts and the first part used for simulation/training of the network model, while the second is used for validation and testing the model (Figure 5). While considering the data for modelling, the data is randomly sampled along with noisy synthetic data with 5% Gaussian noise added to make the model more general. The MANFIS used has 3 inputs, \(F_1, F_2, F_3\) (Ref. eq.8) and two outputs \(R, Z\), radius and depth of the cavity respectively. This MANFIS is a combination of two ANFIS’s each with 3 inputs and one output. ANFIS adjusts the membership function parameters, which specify their shapes and partition of the membership function. The network model is well simulated to learn from training which require only 8 epochs to show a RMSE error (Root Mean Square Error) below 0.0001 (Figure 6). The model validation was also processed by the input vectors from input/output data sets for which the FIS was not trained, to see how well it predicts the corresponding data set output values. Another type of validation data set is referred to as the checking data set and is used to control the potential for the model over-fitting the data. When checking data is presented to MANFIS, the FIS model with the minimum checking data model error is selected.

Figure 5 Schematic diagram of preparing training/validation data

Figure 6 ANFIS Network RMS error versus epochs.

Test of MANFIS for synthetic-noisy data and for real data

After the training the designed MANFIS model was tested with noisy data with 20% of random noise. The results are represented in table 1 which shows the robustness of the method in presence of noise. The MANFIS model was also tested for real data. The gravity data was measured at a site selected in Free Port Bahamas (Styles et al. 2005) and the residual gravity anomaly is shown in Figure 7. We compared the depth estimation of MANFIS with the MLP (Multi Layer Perceptron) neural network method. Five principal profiles were chosen (black lines in Figure 7) were we calculated the features \(F_1,F_2,F_3\) which were applied to the trained MANFIS as inputs. The MANFIS estimated parameters are compared with MLP results (Table 2) and are near to the real depth and radius of the confirmed cavities. The model works well with any data sets that fall within the limits of training range. Otherwise, the same model can be trained with a new data set, to account for different possible environments and to make model more general.

Conclusions

We propose a new method for intelligent interpretation of gravity data to estimate depth via MANFIS which has been tested for synthetic data of models of spheres and cylinders in the presence of noise and the results are remarkably resilient to the presence of noise. The method was also tested for Bahamas microgravity data and the results agree well with the observed drilled values.
Table 1 Outputs of MANFIS in presence of 20% noise.

<table>
<thead>
<tr>
<th>Horizontal Cylinder</th>
<th>Sphere</th>
<th>Outputs of MANFIS(3,5,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(m)</td>
<td>Z(m)</td>
<td>R(m)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.17</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2.15</td>
</tr>
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<tr>
<td>4</td>
<td>8</td>
<td>4.15</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>5.13</td>
</tr>
</tbody>
</table>

Table 2 Interpreted cavity depth and radius through MANFIS method for the selected principle profiles of Grand Bahamas site, in comparison with borehole results and MLP network method.

References