Shape factor and depth estimation of microgravity anomalies via combination of Artificial Neural Networks and Fuzzy Rules Based System (FRBS) in order to detect and characterization of cavities, Case study: Kalgorlie Gold Mine, Australia

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Abstract:
In this paper a new method is presented for shape factor estimation of microgravity anomalies. The method is based on training of a designed neural network (NN) with a vast training data of several objects with different shapes as: Vertical Cylinder, Sphere and Horizontal Cylinder. The input of the NN is the residual anomaly of the selected principle profile and the output is the depth and shape factor of the related object. To extract the most probable shape of the object, three main If-then fuzzy rules are used and the membership degree of the shape to the fuzzy set of {near to: Vertical Cylinder, Sphere and Horizontal Cylinder} is achieved. The method is tested for synthetic data and also real data measured on a mining shaft in Kalgorlie Gold Mine in Australia. The most advantage of the method is its ability to specify how much the gravity target is near to the estimated shape and depth with no need to pre-assumption about its shape.

Keywords: Microgravity, Neural Networks, Fuzzy Rules, Shape factor

Introduction
The aim of gravity interpretation is to discover how masses producing a given gravity anomaly are distributed. Although great efforts have been devoted to this topic, it is clear that no ideal theoretical or practical solution to solve the problem completely exist. In applied geophysics, frequently used elementary shapes for cavities, namely sphere, horizontal cylinders and vertical cylinders, are considered accurate enough for representing bodies (Grêt et al. 2000). Several methods have been developed to interpret gravity anomalies assuming simple causative bodies; these methods include depth rules (Smith 1959), Fourier transformation (Odegard and Berg 1965; Sharma and Geldrat 1968), Euler deconvolution (Thompson 1982; Reid 1990), Mellin transforms (Mohan et al. 1986; Shaw and Agarwal 1990), least-squares minimization approaches (Gupta 1983; Abdelrahman et al. 2001), methods of inverting gravity data to determine model parameters (Li and Oldenburg 1998; Li and Chouteau, 1998; Boulanger and Chouteau 2001). Recently, considerable effort has been devoted to the use soft computing approaches for automatic interpretation of gravity data using artificial neural networks. Elawadi et al. (2001) used back propagation neural network for detection of cavities from gravity data. Salem et al. (2003) used
Hopfield neural network for imaging subsurface cavities from microgravity data. Osman et al. (2007) used Forced Neural Networks for forward modeling of gravity anomaly profile. In all the mentioned methods via neural computation a pre-assumption about the cavity shape is considered and then the neural network is trained with that of supposed shape. In this paper we have combined neural networks with fuzzy sets and if-then fuzzy rules to achieve a method through which no pre-assumption about the cavity shape is needed and also it will be able to estimate from the calculated shape factor that how much is the gravity source near to sphere, horizontal cylinder or vertical cylinder via some if-then fuzzy rules.

OVERVIEW OF ARTIFICIAL NEURAL NETWORKS

Artificial neural networks (ANNs) are a form of artificial intelligence which attempt to mimic the behavior of the human brain and nervous system. A comprehensive description of ANNs is beyond the scope of this paper. A typical structure of ANNs consists of a number of processing elements (PEs), or nodes, that are usually arranged in layers: an input layer, an output layer and one or more hidden layers (Figure 1).

The input from each PE in the previous layer \( (x_i) \) is multiplied by an adjustable connection weight \( (w_{ji}) \). At each PE, the weighted input signals are summed and a threshold value \( (\theta_j) \) is added. This combined input \( (I_j) \) is then passed through a non-linear transfer function \( (f(\cdot)) \) to produce the output of the PE \( (y_j) \). The output of one PE provides the input to the PEs in the next layer. This process is summarized in Equations 1 and 2 and illustrated in Figure 1.

\[
I_j = \sum w_{ji} x_i + \theta_j \quad \text{summation} \quad (1)
\]

\[
y_j = f(I_j) \quad \text{transfer} \quad (2)
\]

The propagation of information in ANNs starts at the input layer where the input data are presented. The network adjusts its weights on the presentation of a training data set and uses a learning rule to find a set of weights that will produce the input/output mapping that has the smallest possible error. This process is called “learning” or “training”. Once the training phase of the model has been successfully accomplished, the performance of the trained model has to be validated using an independent testing set.

Fuzzy Sets Principles

Fuzzy logic was invented by Zadeh in 1965 as an extension of Boolean logic. While classical logic assigns to a variable either the value 1 for “true” or the value “0” for “false”, fuzzy logic allows one to assign to a variable any value in the interval \([0,1]\). This extension is motivated by the observation that humans often think and communicate in a vague and uncertain way, partly because of insufficient information, partly due to human brains nature (Nelles, 2001). Fuzzy logic approach can help engineers and researchers to tackle uncertainty, and to handle imprecise information in a complex
situation (Nikravesh and Aminzadeh, 2003). During the past several years, the successful application of fuzzy logic for solving complex problems subject to uncertainty has greatly increased and today fuzzy logic plays an important role in various engineering disciplines. The uncertain, fuzzy, and linguistic nature of geophysical and geological data makes it a good candidate for interpretation through fuzzy set theory. The main advantage of this technique is in combining the quantitative data and qualitative information and subjective observation. The imprecise nature of the information available for interpretation makes fuzzy sets theory an appropriate tool to utilize (Nikravesh and Aminzadeh, 2003). In crisp sets if we use Membership Function (MF, characteristic function or discrimination function) we can represent whether an element $x$ is involved in a set $A$ or not. In this state we define $MF\, \mu_A$ for a set $A$ such as:

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if } x \in A \\ 0 & \text{if and only if } x \notin A \end{cases}$$  \quad (3)

We can say that the function $\mu_A$ maps the elements in the universal set $X$ to the set $\{0, 1\}$:

$$\mu_A: X \rightarrow \{0, 1\}$$  \quad (4)

In fuzzy sets, each element is mapped to $[0, 1]$ by membership function, where $[0, 1]$ means real numbers between ‘0’ and ‘1’ (including 0, 1). Consequently, fuzzy set is “vague boundary set” comparing with crisp set:

$$\mu_A: X \rightarrow [0,1]$$

As an example used in this study, a cavity can be a member of the set $C=\{\text{cavity} \mid \text{cavity shape is close to sphere}\}$ with a membership degree $\mu_C=0.7$, it means that the cavity is not exactly sphere but near to sphere with a membership degree of 0.7.

**Extracting suitable Fuzzy sets and Fuzzy rules for Cavities Shape estimation**

The gravity effect of a cavity depends on its size, depth and shape. Abdelrahman (2001) presented the analytical formulation for gravity anomaly of objects with different shapes as in equation (5):

$$g(x) = \frac{A}{(x^2 + z^2)^{\frac{3}{2}}}$$  \quad (5)

Where $Z$ is depth of the subsurface object (Fig. 2), $x$ is the horizontal distance over the measured point, $A$ is the amplitude factor, which depends on the size and density contrast of the object, and $q$ is the shape factor which depends on the shape of the object. The values of amplitude factor ($A$) and shape factor ($q$) are listed in Table 1.

<table>
<thead>
<tr>
<th>Shape</th>
<th>$q$ (Shape factor)</th>
<th>$A$ (Amplitude factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>1.5</td>
<td>$\frac{4}{3} \pi G \rho R^3$</td>
</tr>
<tr>
<td>Horizontal Cylinder</td>
<td>1</td>
<td>$2\pi G \rho R^3$</td>
</tr>
<tr>
<td>Vertical Cylinder</td>
<td>0.5</td>
<td>$\pi G \rho R^3$</td>
</tr>
</tbody>
</table>

Table 1. The values of amplitude factor ($A$) and shape factor ($q$) in equation 4, for different shapes of objects (where $R$ is the radius of the object as shown in Fig. 2, $G$ is the universal gravity constant and $\rho$ is the density contrast.)
As mentioned above and in Table 1, $q$ is the shape factor which describes the source geometry and is equal to 0.5, 1 or 1.5 for horizontal cylinder, vertical cylinder and sphere, respectively. In practice, the value of the shape factor is not exactly equal to the mentioned values because of the nature of the real natural cavities which are mostly not exactly but almost near to sphere or cylinder (vertical or horizontal), see Figure 3.

After normalizing the gravity data by dividing the residual gravity by its value at $x=0$ (where the residual gravity has its maximum amplitude), the residual values obtained will only depend on the depth and shape factor of the object. The details are deduced from equation 4 as follows (Abdelrahman 2001):

$$g_n(x) = \frac{g(x)}{g_o(x)} = \frac{g(x)}{g(x)|_{x=0}} = \left(\frac{z^2}{x^2 + z^2}\right)^q$$

Where $g_n$ is the normalized residual gravity value. Note that for cavities the density contrasts are negative and both $g(x)$ and $g_o(x)$ are also negative, and consequently the normalized residual gravity will be positive.

In real cases, regard to the most probable shapes of the cavities, they are classified into three main fuzzy sets:

$\tilde{A}_1 = \{\text{cavity} | \text{the cavity shape is near to sphere}\}$

$\tilde{A}_2 = \{\text{cavity} | \text{the cavity shape is near to vertical cylinder}\}$

$\tilde{A}_3 = \{\text{cavity} | \text{the cavity shape is near to horizontal cylinder}\}$

Also to extract the shape of a cavity from the calculated shape factor via its gravity effect, three main if-then Fuzzy Rules are considered:

Rule 1: If $q$ is near to 1.5 then the cavity shape is near to sphere

Rule 2: If $q$ is near to 1 then the cavity shape is near to vertical cylinder

Rule 3: If $q$ is near to 0.5 then the cavity shape is near to horizontal cylinder

Other secondary fuzzy rules can be deduced using the extension principle (see Appendix A). So from the above main fuzzy if-then rules the below extended fuzzy rules can be extracted:
Rule 1-1: If \( q \) is very near to 1.5 then the cavity shape is very near to sphere.
Rule 1-2: If \( q \) is to some extent near to 1.5 then the cavity shape is to some extent near to sphere.
Rule 2-1: If \( q \) is very near to 1 then the cavity shape is very near to vertical cylinder.
Rule 2-2: If \( q \) is to some extent near to 1 then the cavity shape is to some extent near to vertical cylinder.
Rule 3-1: If \( q \) is very near to 0.5 then the cavity shape is very near to horizontal cylinder.
Rule 3-2: If \( q \) is to some extent near to 0.5 then the cavity shape is to some extent near to horizontal cylinder.

The three main fuzzy rules can be rewritten via mathematical sets language as:

If \( q \approx 1.5 \) then the cavity \( \in \tilde{A}_1 \)
If \( q \approx 1 \) then the cavity \( \in \tilde{A}_2 \)
If \( q \approx 0.5 \) then the cavity \( \in \tilde{A}_3 \)

Where the sign " \( \approx \) " means near to or approximately equal.

So we need to define fuzzy sets for: \( q \) near to 0.5, \( q \) near to 1 and \( q \) near to 1.5. In general we require to define fuzzy sets \( \tilde{A}_i = \{ \text{real number near to } q_i \} \), where \( q_i \) is equal to 0.5, 1 or 1.5 respectively for \( i = 1, 2, 3 \). The boundary for set “real number near to \( q_0 \)” is pretty ambiguous. There are lots of MFs like: Bell-Shaped, Triangular-shaped, Trapezoidal-shaped, Gaussian or Exponential-shaped, Z-shaped, Pi-shaped, S-shaped, Parabolic-shaped, Sigmoid, etc. We found that the Generalized Bell-shaped, Z-shaped, Pi-shaped, S-shaped and Trapezoidal-shaped Membership Functions are not good enough for our purpose because in this study the only point that the membership degree equals to one is the state that the cavity shape is exactly one of the probable shapes of sphere, horizontal cylinder or vertical cylinder. On the other hand for \( \mu_{A_i}(q) \) there is only one point that the \( MF=1 \) so the useful MF’s in this way are triangular-shaped, Gaussian shaped and Parabolic-shaped. Consequently, the possibility of real number \( q \) to be a member of prescribed set can be defined by the following common membership functions:

A. Parabolic-shaped MF:

\[
\mu_{A_i}(q) = \frac{1}{1+k(q-q_0)^2} \quad \text{where } k \text{ is a constant positive variable}; \quad (7)
\]

An example of this type of MF’s is illustrated in figure 4.

![Fig4. A Parabolic-shaped MF with q0=1 and k=100.](image)

B. Triangular-shaped MF:

\[
\mu_{A_i}(q) = \begin{cases} 
1 - \frac{|q-q_0|}{d} & q_0 - d < q < q_0 + d \\
0 & \text{otherwise}
\end{cases} \quad \text{where } d \text{ is a constant variable}. \quad (8)
\]
A triangular-shaped MF for fuzzy set ‘real numbers near to 1’ with d=0.2 and q₀=1, is presented in figure 5.

C. Gaussian Membership: The symmetric Gaussian function depends on two parameters q₀ and σ as given by equation 9:

\[ \mu_{A_i}(q) = e^{-\frac{(q-q_0)^2}{2\sigma^2}} = e^{-k(q-q_0)^2}; \]

where \( k \) is a positive constant real number and \( \sigma \) is.

A Gaussian membership function with \( \sigma=0.3 \) (\( K=5.5556 \)), \( q_0=1.5 \) is depicted in figure 6.

For all the above MFs four characterizations are considered which are in agreement with common sense:

1. For real numbers far from \( q_0 \), the membership degree is zero.
2. For \( q=q_0 \) the membership degree takes its maximum value which is equal by one.
3. More far from \( q_0 \) more the membership degree decreases.
4. For \( q<q_0 \) the MF is increasing and for \( q>q_0 \) the MF is decreasing.

In order to find the optimum MF for \( q \) we tested the mentioned MF’s with an algorithm base on a trial and error procedure with different constants for the above MF’s and finally found that the best MF type for our study is Gaussian MF with an optimized \( \sigma \) (for each of \( q_0 \)) listed in Table 3.

The problem then is that i.e. we know from Rule 1 that if \( q \) is near to 1.5 then the cavity shape will be near to sphere but we can’t say that how much is it near to exact sphere? On the other hand, an attempt is to specify membership function for set \( \tilde{A_i} \), from the calculated \( q \). This is also a mapping from the real number (q) space to the cavity’s shape space, so that by knowing the value of \( q \), the MF of the cavity in set \( A_i \) is specified properly. To evaluate the MF, we considered the fact that more the cavity shape similar to i.e. exact sphere more its gravity effect similar to the residual anomaly of the
Shape factor and depth estimation

exact sphere, also with a shape factor near to related exact shape. In order to measure the similarity of
the object anomaly to the Exact-Shapes (ES) anomaly. ES: sphere, vertical cylinder or horizontal
cylinder, its gravity effect is compared with the gravity effect of ES. In this way, three main indexes are
considerable:

Mean Absolute Error: \( MAE = \frac{1}{n} \sum_{i=1}^{n} | g_i(x_i, q_0, z) - \bar{g}_i(x_i, q, z) | \) \hspace{1cm} (10)

Where \( g_i(x_i, q, z) \) and \( \bar{g}_i(x_i, q_0, z) \) are the gravity effects of cavities with depth \( z \) and shape factors \( q, q_0 \)
on the point with horizontal distance from its center, respectively and \( n \) is the total number of gravity
points along the selected profile.

Correlation Coefficient: \( R = \sqrt{\frac{1}{n} \sum_{i=1}^{n} [ g_i(x_i, q_0, z) - \bar{g}_i(x_i, q_0, z) ] [ g_i(x_i, q, z) - \bar{g}_i(x_i, q, z) ]} \) \hspace{1cm} (11)

Where \( \bar{g}_i(x_i, q_0, z) \) is the mean value of \( g_i(x_i, q_0, z) \) equals to \( \frac{1}{n} \sum_{i=1}^{n} g_i(x_i, q_0, z) \) and
\( \bar{g}_i(x_i, q, z) \) is the mean value of \( g_i(x_i, q, z) \) equals to \( \frac{1}{n} \sum_{i=1}^{n} g_i(x_i, q, z) \).

Root-Mean Square Error: \( RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} ( g_i(x_i, q, z) - \bar{g}_i(x_i, q_0, z) )^2} \) \hspace{1cm} (12)

In the present study, we found RMSE to be good enough to evaluate the similarity of the anomaly of the
Exact-Shapes (ES) and Near –Shapes (NS) cavity. For the ES cavity with depth \( z_0 \):
\( g_i(x_i, q, z_0) = g_i(x_i, q_0, z_0) \) \hspace{1cm} : \hspace{1cm} For all \( i=1, 2, ..., n, q_0 \in \{0.5, 1, 1.5\} \)

In this state from equation (12) \( RMSE=0 \) but as the cavity is ES the membership degree \( \mu_{Ai} \) should
be one. Also for the maximum value of the RMSE the membership degree \( \mu_{Ai} \) should be zero. In our study
as a geological prior information the maximum depth for probable cavities is about 15 meters, so
RMSE values for each of fuzzy sets \( \bar{q} \) are calculated for cavities in different depths from \( Z=0.5 \) meter
to \( Z=15 \) meter.

Then the RMSE values are normalized for each set, because for the maximum value of RMSE, the
similarity of the cavity shape to ES reaches to its minimum value and so:
\( \mu_{Ai} = 1 - (RMSE)_h \) \hspace{1cm} (13)
\( RMS_n = \frac{RMSE - RMSE_{min}}{RMSE_{max} - RMSE_{min}} \) \hspace{1cm} (14)

As mentioned in last paragraph \( RMSE_{min} = 0 \), so:
\( RMS_n = \frac{RMSE}{RMSE_{max}} \)

MFs are extracted by equations 12, 13; some of the data produced in order to achieve the related
fuzzy membership-plane are listed in Tables 2, 3, 4 respectively.

The Fuzzy Rule Based System (FRBS) for depth and shape estimation with related
membership degree

In a general view the algorithm used in this research is depicted in Fig.7.
Test of the fuzzy rule-based model with real data
As a test for real data we used the microgravity data measured for Calgarlie Gold mine located in west of Australia. The gravity anomaly map is depicted in Figure 8 with the selected profile shown with red-star line.

Fig. 7. Block diagram of the combination of fuzzy rules and neural network in order to shape and depth estimation of cavities.

Fig. 8. Gravity anomaly map of Calgarlie Gold Mine, Australia

Fig. 9. The result of Fuzzy-Rule based system (FRBS) for the chosen profile (red) in comparison with real data.
Table 1. Results of FRBS for the selected gravity profile

<table>
<thead>
<tr>
<th>Selected Gravity Profile</th>
<th>Real Cavity shape</th>
<th>Real Depth</th>
<th>Real shape factor</th>
<th>Estimated Depth using FRBS</th>
<th>Estimated shape factor using FRBS</th>
<th>Relative Error for depth(%)</th>
<th>Relative Error for shape factor(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47900 North Near to Vertical Cylinder</td>
<td>4 m</td>
<td>0.5</td>
<td>3.5</td>
<td>0.43</td>
<td>14.28</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

The Fuzzy Rule Base System (FRBS) in combination with Artificial Neural Networks was used as estimator of depth and shape factor of cavities. The method was tested for real microgravity data of Kalgoorlie Gold Mine in west Australia and the results showed that the estimated values are very near to the actual depth and shape of the cavities.

References:


